How Large are Learning Externalities?*

Seung Mo Choi[1]

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Abstract

The quantitative features of human capital externalities are not fully understood. While static externalities are estimated in some studies, learning externalities remain mostly unestimated. By calibrating a growth model, this paper provides an estimate of learning externalities. The calibration uses an equilibrium condition that equates private returns on physical capital and human capital. The results suggest that sizable learning externalities exist, even in a conservative set-up. The estimated social rate of return on human capital is 9.0 percent, compared to the private rate of return, 6.6 percent. Therefore, human capital externalities are an important source of economic growth.

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1. Introduction

The main finding of growth accounting, whether applied to a single economy over time or to a cross-section of economies, is that differences in measured inputs account for only a small part of the difference in measured output (see Barro and Sala-i-Martin 2004, Chap. 10). However, a central assumption underlying most growth accounting exercises is that social and private returns are equal. This assumption is no longer valid if external effects are present in production.

A prime example is the case of learning externalities. Suppose that the individual production of human capital depends not only on individual inputs but also on the overall level of knowledge in the economy. Since more knowledge is available, today's typical college graduates will acquire more human capital than a generation ago. Hence, the count of equivalent college graduates, which is commonly used in growth accounting, may underestimate the actual accumulation of human capital. If learning externalities are important, part of the unexplained residual in growth accounting can be attributed to a contribution from human capital accumulation.

Conceptually, human capital externalities may appear in the production of human capital as learning (or dynamic) externalities or in the production of goods as static externalities.[2] As I review the literature below, the studies on cross-country schooling data may capture both forms of externalities. However, these studies suffer from empirical issues and the results are often inconclusive. On the other hand, micro-level studies, which typically use Mincer regressions to explain cross-sectional earnings of workers with factors such as schooling, work experience, and human capital externalities, suffer from fewer empirical issues. However, they tend to focus only on static externalities. In short, improvements in the estimation of both forms of human capital externalities, especially of learning externalities, are needed.

Are learning externalities substantively large? The goal of this paper is to estimate

[2] They may also appear as non-production externalities, for example, by reducing crime rates, by increasing civic participation, and by enhancing political stability. Davies (2003) provides a survey on non-production externalities. However, this paper focuses on the externalities in production sectors.

learning externalities through the calibration of a deterministic, closed-economy, semi-endogenous growth model based on the U.S. macro-level data. In the model, human capital accumulation and exogenous productivity growth are two engines of economic growth. The representative consumer allocates her human capital into two production sectors, goods and human capital. Goods production takes a Cobb-Douglas form with two inputs, the services of physical capital and of human capital. Human capital production follows the same general form, but the two inputs are the services of the consumer's own human capital and the per-capita human capital holding in the economy (taken as exogenous by the consumer). While the model is similar to Tamura (1991) and Lucas (2004), this paper explores quantitative rather than theoretical issues.

To empirically identify learning externalities, I need to identify the parameters in the human capital production function. To do this, I focus on the marginal product and the total product of human capital. An equilibrium condition is that the (private) marginal product of human capital is equal to the marginal product of physical capital.[3] Also, from the human capital production function, the total product of human capital is equal to a function of the growth rate of per-capita output.

The calibration results suggest that the power parameter for per-capita human capital in the economy is substantially greater than zero. Hence, learning externalities are an important input in human capital production. The result indicates that the contribution of learning externalities to human capital production is worth approximately 20 percent of GDP. The social rate of return on human capital is 9.0 percent, compared to the private rate of return, 6.6 percent.

The remainder of this section discusses the related literature and the calibration approach. Section 2 describes the model. Section 3 calibrates the model. Section 4 connects my calibration to the empirical labor literature. Section 5 provides some concluding remarks.

[3] This condition is also used by Parente and Prescott (2000, p. 60) in a different model.

1.1. Related Literature

Human capital externalities often improve the fit of a quantitative model (e.g., Tamura 2006). This indirectly suggests sizable human capital externalities. Some papers focus on estimating them. There are two main methods.

The first method is a micro-level Mincer regression designed to capture static externalities. With this approach, explanatory variables representing externalities are added to more standard variables such as schooling and work experience. Average schooling in communities is an example of the variables used to represent static externalities. Pioneering studies include Rauch (1993) and Heckman, Layne-Farrar and Todd (1996). Highly cited studies include Acemoglu and Angrist (2000), Moretti (2004), and Ciccone and Peri (2006). While Acemoglu and Angrist (2000) and Ciccone and Peri (2006) find insignificant or negligible externalities, Moretti (2004) reports that they are significant. Recent studies tend to support Moretti (2004). Dalmazzo and de Blasio (2007) use rent data in the analysis of a Mincer regression and find significant externalities. Liu (2007) and Muravyev (2008) analyze data from China and Russia, respectively, and find significant externalities. In sum, the literature is inconclusive, but recent studies support significant externalities.

Unfortunately, Mincer regressions like these do not capture learning externalities. To capture learning externalities, a Mincer regression should include explanatory variables representing the environments that an individual worker has been exposed to during his or her human capital accumulation. Partly owing to data limitations, few studies estimate learning externalities. Borjas (1992, 1995) assumes a human capital production function that is close to this paper's and finds that average skills of the ethnic group in parents' generation are an important determinant of skills of today's generation, supporting significant learning externalities.[4] However, the size of learning externalities at a country level remains undocumented in a conclusive way, which motivates this study.

[4] In addition, Foster and Rosenzweig (1995) find that farmers are more likely to apply a new technology when neighbors have already adopted it. Other studies estimate peer effects in the classroom (see, for example, Hoxby 2000).

The second method to estimate human capital externalities is a cross-country growth regression. The estimated returns to schooling at the country level are considered social returns. Researchers compare them to the private returns at the individual level, estimated in a micro-level Mincer regression. Both forms of human capital externalities are captured by this growth regression. However, empirical problems such as measurement errors, reverse causality, and omitted variables make this type of analysis difficult (see Krueger and Lindahl 2001). Partly due to the empirical problems, existing studies tend to be inconclusive. Barro's (1991) growth regression implies that social returns can be twice as high as private returns (see Davies 2003). McMahon (1999) also reports significant excess returns. However, Heckman and Klenow (1997) and Topel (1999) do not find clear evidence for or against sizable externalities.

2. The Model

2.1. Model Description

The closed economy has an infinitely-lived representative consumer who also serves as a producer. Goods production follows a constant-returns-to-scale Cobb-Douglas function with two inputs, the services of physical capital and human capital:

$$Y(t) = K(t)^{\alpha}[A(t)[1 - u(t)]h(t)L(t)]^{1-\alpha},$$

for $0 < \alpha < 1$, where $Y(t)$ is physical goods produced, $K(t)$ is physical capital stock, $A(t)$ is labor-augmenting productivity which grows exogenously at $g_A \geq 0$, $1 - u(t)$ is a fraction of human capital stock devoted to goods production, $h(t)$ is per-capita human capital stock, and $L(t)$ is the population which grows exogenously at $g_L \geq 0$. The law of motion of physical capital is $\dot{K}(t) = I(t) - \delta_K K(t)$, where $I(t)$ is the physical goods accumulated, and $0 \leq \delta_K < 1$ is a depreciation rate of physical capital. All $Y(t)$, $K(t)$ and $I(t)$ are in units of physical goods. The law of motion of per-capita human capital is

$$\dot{h}(t) = B(t)\,[u(t)h(t)]^{\phi}\,\overline{h}(t)^{\theta} - (\delta_h + g_L)h(t), \tag{1}$$

for $0 < \phi < 1$, $0 < \theta < 1$, $\phi + \theta \leq 1$ (which enables the model to have a balanced growth path) and $0 \leq \delta_h < 1$. Here, $B(t)$ is the productivity of human capital production which grows exogenously at $g_B \geq 0$. The first term of the right-hand side is the individual human capital production function, which is Cobb-Douglas with two inputs: (i) services of human capital, where $u(t)$ is, of course, a fraction of human capital stock devoted to human capital production, and (ii) learning externalities, measured by the average human capital stock in the economy, denoted by $\overline{h}(t)$. The path of $\{\overline{h}(t)\}$ is taken as exogenous by the consumer. Clearly, the power on learning externalities, θ, is a key parameter to calibrate. Each period, a fraction δ_h of the human capital stock depreciates. Since $h(t)$ is in a per-capita term, its accumulation is adjusted by g_L.

I discuss four features of the model. First, there is a single, infinitely-lived representative consumer, but the population is allowed to grow, as in Lucas (1988). An interpretation is that the economy consists of many finitely-lived consumers with altruism. The infinitely-lived representative consumer in the model is interpreted as an average finitely-lived consumer in reality (see Barro and Sala-i-Martin 2004, p. 86, for further discussion). Alternatively one could introduce an overlapping-generations model, but this would complicate the analysis without significant gains (see Barro and Sala-i-Martin 2004, p. 178).

Second, the model is semi-endogenous, featuring two engines, human capital accumulation and exogenous productivity growth. I assume that human capital accumulation contributes, at least partly, to output growth on the balanced growth path.[5] An alternative specification would be to consider an exogenous growth model, in which $A(t)$ is a sole engine of growth and $h(t)$ determines a relative output level.[6]

Third, the model disregards physical inputs in human capital production. Adding physical inputs to (1) complicates the analysis without changing the calibration results

[5] As I discuss later, Jorgenson and Yip (2001) and Rangazas (2005), among others, suggest that there is a contribution from human capital accumulation to output growth.

[6] As it becomes clear later, this alternative specification would require additional data observations for calibration. The differences in human capital production activities, across economies or over periods, can be useful.

substantially. I proceed with (1), but later discuss how introducing physical inputs in human capital production affects the benchmark result.

Fourth, $B(t)$ is allowed to grow. By construction, this leaves less room for learning externalities because $B(t)$ growth becomes an additional source of human capital accumulation. In other words, this set-up provides a conservative estimate of learning externalities.

The resource constraint is

$$\begin{aligned}(2)\qquad Y(t) &= I(t) + C(t) + \tau_C C(t) + \tau_K r(t)K(t) \\ &\quad + \tau_L w(t)A(t)\left[1-u(t)\right]h(t)L(t) - sw(t)A(t)u(t)h(t)L(t),\end{aligned}$$

where $C(t)$ is the physical goods consumed, and, $0 \leq \tau_C < 1$, $0 \leq \tau_K < 1$ and $0 \leq \tau_L < 1$ are constant tax rates on consumption, physical capital income, and human capital income (or labor income) in goods production. Also, $r(t) \equiv \alpha K(t)^{\alpha-1}[A(t)\left[1-u(t)\right]h(t)L(t)]^{1-\alpha}$ is the interest rate before taxes are imposed. Similarly, $w(t) \equiv (1-\alpha)K(t)^{\alpha}[A(t)[1-u(t)]h(t)L(t)]^{-\alpha}$ is the rental rate per effective unit of human capital in goods production, augmented by exogenous productivity $A(t)$, before taxes are imposed. Thus, the rental rate per human capital becomes $w(t)A(t)$. I define $w(t)$ in this way to keep $w(t)$ constant on the balanced growth path. Finally, $sw(t)$ is a subsidy provided per effective human capital unit in human capital production. The remaining government revenues after the subsidies are paid are thrown away.

Using the properties of the production function, (2) can be rearranged as

$$Y(t) = I(t) + (1+\tau_C)C(t) + \left[\alpha\tau_K + (1-\alpha)\tau_L - \frac{(1-\alpha)su(t)}{1-u(t)}\right]Y(t).$$

Obviously, the after-tax physical capital income is $(1-\tau_K)\alpha Y(t)$. The after-tax human capital income in goods production is $(1-\tau_L)(1-\alpha)Y(t)$. Notice that the human capital in human capital production does not pay labor income taxes. An interpretation is that the subsidy is in net term after labor income taxes are paid. Then, each unit of human capital earns the value of the marginal product in human capital production, plus the subsidy, $sw(t)A(t)$. The government revenue after subsidies, which are thrown away, become $\tau_C C(t) + \left[\alpha\tau_K + (1-\alpha)\tau_L - (1-\alpha)s\frac{u(t)}{1-u(t)}\right] Y(t)$. This is assumed to be positive throughout this paper.

Finally, the representative consumer has preferences with a constant-relative-risk-aversion parameter, σ, where $\sigma > 0$ and $\sigma \neq 1$. As $\sigma \longrightarrow 1$, the preferences approach to the log utility. An equilibrium of this economy is defined as follows.

Definition 1 *An equilibrium is a path of* $\{C(t), h(t), \overline{h}(t), K(t), u(t), Y(t)\}_{t=0}^{\infty}$ *such that*

(a) the representative consumer solves

$$\max_{\{C(t),\, u(t)\}_{t=0}^{\infty}} \int_0^\infty e^{-\rho t} \frac{(C(t)/L(t))^{1-\sigma}}{1-\sigma} L(t)dt, \tag{3}$$

for $\sigma > 0$ *and* $\sigma \neq 1$, *satisfying* $C(t) > 0$ *and* $0 < u(t) < 1$, *subject to*

$$\dot{K}(t) = \left[1 - \alpha\tau_K - (1-\alpha)\tau_L + \frac{(1-\alpha)su(t)}{1-u(t)}\right] Y(t) - (1+\tau_C)C(t) - \delta_K K(t), \tag{4}$$

$$\dot{h}(t) = B(t)\left[u(t)h(t)\right]^{\phi} \overline{h}(t)^{\theta} - (\delta_h + g_L)h(t), \tag{5}$$

for all $t > 0$, *with* $Y(t) = K(t)^{\alpha}[A(t)[1-u(t)]h(t)L(t)]^{1-\alpha}$, $\dot{A}(t) = g_A A(t)$, $\dot{B}(t) = g_B B(t)$ *and* $\dot{L}(t) = g_L L(t)$, *given* $(K(0), h(0), A(0), L(0))$ *and* $\{\overline{h}(t)\}$ *for* $t \geq 0$, *for* $0 < \alpha < 1$, $0 \leq \tau_K < 1$, $0 \leq \tau_L < 1$, $s \geq 0$, $0 \leq \tau_C < 1$, $0 \leq \delta_K < 1$, $\phi > 0$, $\theta > 0$, $\phi + \theta < 1$, $0 \leq \delta_h < 1$, $g_L \geq 0$, $g_A \geq 0$, $g_B \geq 0$, $K(0) > 0$, $h(0) > 0$, $A(0) > 0$, $L(0) > 0$ *and* $\overline{h}(t) > 0$ *for all* $t > 0$;

(b) a condition $\overline{h}(t) = h(t)$ *holds.*

2.2. Equilibrium Conditions on the Balanced Growth Path

I focus on a balanced growth path of an equilibrium in which (i) all variables grow at constant rates (or stay at constant levels) and (ii) $u(t)$ is constant. The second restriction addresses two important concerns: If $u(t)$ had a strictly positive growth rate, then it would eventually violate $u(t) < 1$. If $u(t)$ had a strictly negative growth rate, then it would converge to 0, which is not consistent with empirical fact. Appendix 1 solves for the equilibrium on the balanced growth path. This equilibrium is unique under certain conditions that are satisfied under the calibrated parameter values.

Here, I economically interpret the conditions in the equilibrium on the balanced growth path and manipulate them as a convenient form for the calibration. Time indicators are omitted unless required. Throughout this paper, g_X denotes a growth rate of a variable X on the balanced growth path. Then, the growth rates are related as

$$g_Y = g_C = g_K = g_A + g_L + g_h, \tag{6}$$

$$g_B = (1 - \phi - \theta) g_h. \tag{7}$$

The two laws of motions of physical and human capital imply

$$g_K + \delta_K = i/(K/Y), \tag{8}$$

$$g_C - g_A = B_0 h_0^{\phi+\theta-1} u^{\phi} - \delta_h, \tag{9}$$

where $i \equiv (Y - C)/Y$ is the investment-output ratio. For convenience, $B_0 \equiv B(0)$ and $h_0 \equiv h(0)$ to fix a specific period for B and h. At this period 0, $A(0)$ is normalized to 1 without loss of generality. Since K and Y grow at the same rate, (K/Y) is constant. The first-order conditions are

$$\rho + \sigma (g_C - g_L) = \left[1 - \alpha\tau_K - (1-\alpha)\tau_L + \frac{(1-\alpha)su}{1-u}\right] r - \delta_K \tag{10}$$

$$= \left[1 - \alpha\tau_K - (1-\alpha)\tau_L + \frac{(1-\alpha)su}{1-u}\right] \frac{\alpha}{K/Y} - \delta_K \tag{11}$$

$$= \frac{1 - \alpha\tau_K - (1-\alpha)\tau_L - \frac{\alpha su}{1-u}}{1 - \alpha\tau_K - (1-\alpha)\tau_L - \frac{\alpha su}{1-u} - s} \phi B_0 h_0^{\phi+\theta-1} u^{\phi-1} - \delta_h + g_A \tag{12}$$

$$= \left[1 - \alpha\tau_K - (1-\alpha)\tau_L - \frac{\alpha su}{1-u}\right] \frac{(1-\alpha)(K/Y)^{\alpha/(1-\alpha)}}{V_0} - \delta_h + g_A \tag{13}$$

Define $V(t)$ to be a unit value of human capital in units of physical goods. In (13), $V_0 \equiv V(0)$. Equation (10) is an Euler equation, which relates the after-tax-and-subsidy interest rate after depreciation to per-capita consumption growth. The remaining three equations, (11), (12) and (13), equate the rates of returns on the following four types of investments:

1. Lending physical capital to a rental market: The right-hand side of (10) is the after-tax-and-subsidy interest rate in a rental market, after depreciation (δ_K). The term in squared brackets reflects the distortions caused by taxes and subsidies. Notice that this term already appeared in (4).

2. Providing the services of physical capital to goods production: The right-hand side of (11) is the after-tax-and-subsidy marginal product of physical capital in goods production, after depreciation (δ_K).

3. Providing the services of human capital to human capital production: The right-hand side of (12) is the after-tax-and-subsidy marginal product of human capital in human capital production, after depreciation (δ_h) and exogenous productivity growth (g_A). To directly obtain this marginal product without solving for the equilibrium, differentiate $B\,(uh)^{\phi}\,\overline{h}^{\theta}$ with respect to uh and impose $\overline{h} = h$. The term in squared brackets reflects the distortions caused by taxes and subsidies. If $s = 0$, it collapses to 1.

It is important to see that there is an additional term, g_A, in (12), and not in (11). This term comes from the fact that the two marginal products in (11) and (12) are measured in different units. That is, g_A here reflects a value increase of a unit of human capital during one period. Hence, the returns on human capital consist of the marginal product (after depreciation) and the gain in value.

4. Providing the services of human capital to goods production: The right-hand side of (13) is the after-tax-and-subsidy marginal product of human capital in goods production, after depreciation (δ_h) and exogenous productivity growth (g_A). To see this more clearly, write (10) and (13) in a more familiar form in asset pricing as

$$(14)\qquad \begin{aligned}&(r^* - \delta_K)V_0\\ &= \left[1 - \alpha\tau_K - (1-\alpha)\tau_L - \frac{\alpha s u}{1-u}\right](1-\alpha)(K/Y)^{\alpha/(1-\alpha)} + (g_A - \delta_h)V_0,\end{aligned}$$

where $r^* \equiv \left[1 - \alpha\tau_K - (1-\alpha)\tau_L + \frac{(1-\alpha)su}{1-u}\right] r$ is the after-tax-and-subsidy interest rate. This equation implies that one unit of human capital with an initial value V_0 delivers $\left[1 - \alpha\tau_K - (1-\alpha)\tau_L - \frac{\alpha su}{1-u}\right](1-\alpha)(K/Y)^{\alpha/(1-\alpha)}$ (after-tax-and-subsidy marginal product) as a dividend. It also has a value increase by $(g_A - \delta_h)V_0$, through depreciation and exogenous productivity growth. That is, after one period, the owner of this original unit has only $e^{-\delta_h}$ units of human capital due to depreciation, but each unit is now worth $e^{g_A}V_0$ due to productivity growth. This interpretation still holds for any period t in general, by multiplying both sides of (14) by $e^{g_A t}$ and interpreting $V(t) = e^{g_A t}V_0$ as period-t unit value of human capital. That is, I can write

$$(r^* - \delta_K)V = \left[1 - \alpha\tau_K - (1-\alpha)\tau_L - \frac{\alpha s u}{1-u}\right](1-\alpha)(K/Y)^{\alpha/(1-\alpha)} - \delta_h V + \dot{V}$$

for an equivalent interpretation at period t. Notice that this interpretation is consistent with the previous discussion (regarding (12)) that g_A reflects an increase in the unit value of human capital.

The economy is characterized by six equations, (8), (9), (10), (11), (12), and (13), all in terms of constants. In addition, the relationships among growth rates of key variables are expressed in (6), (7), and $g_V = g_A$. I am now ready to calibrate the model.

3. Calibration

3.1. Matching Data to the Model

I use U.S. data, over the span of 1960-2005 whenever possible. Many observations in National Income and Product Accounts (NIPA) start in 1960. The year 2005 was the most recent year when the NIPA data were saved for this study.

Goods production, $Y(t)$, does not coincide with GDP. This is because GDP includes aspects of the human capital production, such as education or scientific research and development (R&D). The objective of these activities is to accumulate knowledge rather than to produce physical goods. Hence, I first exclude them from GDP to measure $Y(t)$.

Human capital income in educational services accounts for 10.3 percent of human capital income of all occupations, in 2005, according to the Occupational Employment Statistics published by the Bureau of Labor Statistics. Observations are not available for the entire span of 1960-2005. However, I conjecture that the share had increased steadily, based on the observation that total educational expenditures relative to GDP increased by 0.6 percent per year over the period 1960-2005.[7] Applying the same growth rate, a reasonable measurement for human capital income in educational services, over 1960-2005, becomes 9.0 percent of total human capital income.

Human capital income in scientific R&D services accounts for 0.8 percent of human capital income of all occupations in 2005. Since the ratio of total R&D expenditures to GDP had been relatively stable,[8] I assume that this ratio was constant over 1960-2005. Therefore, 9.8 percent ($= 0.090 + 0.008$) of human capital income comes from education and scientific R&D.

How large is this compared to GDP? To compute the share of human capital income out of total income, I use "compensation of employees" as the numerator and GDP minus "proprietors' income" minus "taxes on production and imports"[9] as the denominator from NIPA Table 1.12. Then, human capital income is 68 percent of GDP, on average, during the period 1960-2005. Hence, human capital income from educational services and scientific R&D services is 7 percent of GDP ($= 68 \times 0.098$). In other words, goods production is approximately 93 percent of GDP.

From the data, I calibrate the following constants:

1. $\alpha = 0.34$: Physical capital income is 32 percent of GDP from the discussion above. Also, goods production is 93 percent of GDP. Thus, $\alpha = 0.34$ ($= 0.32/0.93$).

2. $g_Y = g_K = g_C = 0.033$: The sum of population growth and growth of "GDP per capita in chained dollars," both of which are reported in NIPA Table 7.1, is 3.3 percent, on average of 1960-2005.

3. $g_L = 0.011$: The population growth from NIPA Table 7.1 is 1.1 percent, on average, over the period 1960-2005.

4. $i = 0.21$: The gross domestic investment in NIPA Table 1.1.5, divided by GDP in NIPA Table 1.1, is 20 percent, on average, over the period 1960-2005. Since goods

[7] Table 25 of the Digest of Education Statistics, published by the U.S. Department of Education, provides total expenditures for education, including the cost of building structures or repaying debts, over 1960-2005.

[8] According to OECD Main Science and Technology Indicators, gross R&D expenditures in real term increased by 3.5% annually from 1983 to 2004. This is close to the annual GDP growth rate, 3.3%.

[9] "Taxes on production and imports" are formerly known as "indirect business taxes."

production is 93 percent of GDP, I have $i = 0.21$ $(= 0.20/0.93)$.

5. $\delta_K(K/Y) = 0.12$: A ratio of the consumption of fixed capital to GDP is 0.11, on average, over the period 1960-2005. according to NIPA Table 1.1. Hence, $\delta_K(K/Y) = 0.12$ $(= 0.11/0.93)$.

6. $\delta_h = 0.035$: Arrazola and de Hevia (2004) estimate an individual-level human capital depreciation through cross-sectional estimation. Their estimates are between 1.0 percent and 1.5 percent; I use the midpoint, 1.3 percent.[10] Assuming a worker's working lifetime is 45 years as conventional, 2.2 percent of the labor force retires every year.[11] The sum of these two, 3.5 percent, is the value assigned to δ_h.[12]

7. $\tau_K = 0.43$, $\tau_L = 0.25$: Mendoza, Razin and Tesar (1994) report that the average tax rates during the period 1965-88 were $\tau_K = 0.43$ and $\tau_L = 0.25$. This period falls in the middle of the period 1960-2005. Extending the time frame does not substantially change these numbers. For example, Carey and Rabesona (2004) report $\tau_K = 0.39$ and $\tau_L = 0.27$ for 1975-2000. McGrattan, Rogerson and Wright (1997) report $\tau_K = 0.53$ and $\tau_L = 0.25$ in 1960-92. Notice that τ_C does not appear in the set of equations calibrated.

8. $u = 0.28$ and $s = 0.15$: Calibrating u and s requires an additional discussion. To begin, I use the observations on subsidies to human capital production to obtain an equation relating u and s. Then, I use the observations on human capital incomes in goods production and human capital production to have another equation, eventually to separate u and s.

(i) $su/(1-u) = 0.06$: First, human capital income in educational services is 9.0 percent

[10] Many recent studies claim a low depreciation rate. Huggett, Ventura and Yaron (2006) assume 1.1% for individual-level depreciation. Heckman, Lochner and Taber (1998) assume 0%.

[11] This measurement assumes that the average human capital stock of a retiree is the same as that of a worker of all age groups. In fact, this assumption is supported by the data. The average retirement age is between 62 and 65 in 1960-2000, according to Gendell (2001). According to Earnings by Occupation and Education, Census of Population, published in three decennial years, 1980, 1990 and 2000, the age-group earnings relative to the national average (of all age groups) are 1.1 at ages 55-64 and 1.0 at ages 65 and above.

[12] Stokey and Rebelo (1995) also follow this approach to calibrate the human capital depreciation rate.

of total human capital income (as discussed above), or equivalently, 6.1 percent of GDP. The 1960-2005 average of the ratio of government expenditure on education to total expenditure on education is 75.0 percent.[13] Hence, the government subsidy to educational services is 4.6 percent ($= 0.061 \times 0.75$) of GDP. Second, human capital income in scientific R&D services is 0.8 percent of total human capital income (as discussed above), or 0.5 percent of GDP. According to Hall and Van Reenen (2000), an R&D subsidy rate is 20 percent. This means that the government spends 0.1 percent of GDP ($= 0.005 \times 0.20/(1 + 0.20)$) as R&D subsidies. Third, I assume that there are no subsidies available for on-the-job training.[14] To conclude, the subsidies are worth 4.7 percent of GDP. After labor income taxes, the net subsidies become 3.5 percent ($= 0.047 \times (1 - 0.25)$) of GDP. In (2), the subsidies correspond to $sw(t)A(t)u(t)h(t)L(t)$, or equivalently, $\frac{(1-\alpha)su}{1-u}Y(t)$. Since $\alpha = 0.34$ and $Y(t)$ is 93 percent of GDP, this implies $su/(1 - u) = 0.06$.

(ii) (income of human capital devoted to human capital production) =16.3 percent of GDP: Recall that human capital income in goods production is 61.1 percent of GDP (as discussed above). To measure human capital income in human capital production, I sum the following three components. (a) Human capital income in educational services and scientific R&D services is 6.7 percent of GDP (discussed above). (b) The foregone human capital income due to schooling is 3.7 percent of GDP (discussed in Appendix 2). (c) The foregone human capital income due to on-the-job training is 5.9 percent of GDP (discussed in Appendix 2). The sum of these three is 16.3 percent of GDP.

(iii) $u = 0.28$ and $s = 0.15$: Each unit of human capital in human capital production

[13] Table 30 of Digest of Education Statistics, published by the U.S. Department of Education, provides the government expenditure on education in state and local levels. Table 25 provides the total expenditure on education.

[14] Although the data are limited, the government subsidies to on-the-job training may not be zero. However, introducing them turns out to increase the estimate of learning externalities. In other words, my benchmark calibration provides a conservative estimate.

earns $\phi B_0 h_0^{\phi+\theta-1} u^{\phi-1} V_0$ (i.e., the value of the marginal product, as interpreted previously regarding the right-hand side of (12)) plus sw (i.e., subsidies), at period 0 in which $A(0) = 1$. However, the equality between the right-hand sides of (12) and (13) implies

$$\phi B_0 h_0^{\phi+\theta-1} u^{\phi-1} V_0 = \left[1 - \alpha\tau_K - (1-\alpha)\tau_L - \frac{\alpha s u}{1-u} - s\right](1-\alpha)(K/Y)^{\alpha/(1-\alpha)},$$

or equivalently, using $w = (1-\alpha)(K/Y)^{\alpha/(1-\alpha)}$,

$$\phi B_0 h_0^{\phi+\theta-1} u^{\phi-1} V_0 + sw = \left[1 - \alpha\tau_K - (1-\alpha)\tau_L - \frac{\alpha s u}{1-u}\right] w.$$

Since this is the income per human capital in human capital production at period 0, the total human capital income in human capital production is equivalent to

$$\left[1 - \alpha\tau_K - (1-\alpha)\tau_L - \frac{\alpha s u}{1-u}\right] w u h_0 L_0$$

at period 0. On the other hand, the total human capital income in goods production is $w(1-u)h_0 L_0$. Therefore,

$$\frac{\left[1 - \alpha\tau_K - (1-\alpha)\tau_L - \frac{\alpha s u}{1-u}\right] w u h_0 L_0}{w(1-u)h_0 L_0} = \left[1 - \alpha\tau_K - (1-\alpha)\tau_L - \frac{\alpha s u}{1-u}\right]\frac{u}{1-u} = \frac{16.3}{61.1}$$

from (ii). Since I know all other parameters including $su/(1-u)$ from (i), I can solve for u. I obtain $u = 0.28$ and $s = 0.15$.

9. $g_h = \frac{1}{3}(g_Y - g_L) = 0.007$: Equation (6) implies $g_Y - g_L = g_h + g_A$.[15] Jorgenson and Yip (2001) report that g_h is almost twice as high as g_A for the United States in 1960-95. Rangazas (2005) also report that g_h is responsible for 1/3 of the U.S. growth.[16] These studies do not consider learning externalities. Thus, if learning externalities are

[15] As in Hall and Jones (1999), I write the growth decomposition with K/Y, rather than with K/L. Mankiw, Romer and Weil (1992) and Klenow and Rodríguez-Clare (1997) also use similar decompositions. On the balanced growth path, K/Y is constant, so this component does not show up in growth decomposition.

[16] Here is a simple way to measure g_h. Assume that schooling is the only form of human capital production. Usual Mincer regressions (without human capital externalities) suggest that an additional

sizable, then g_h would be measured higher. I follow Rangazas (2005) in assuming $g_h = \frac{1}{3}(g_Y - g_L) = 0.007$ as a benchmark. If I assume a higher level of g_h, then the estimate of learning externalities increases. In that sense, the calibration is, again, conservative. Later, I discuss how the results are sensitive to the choice of g_h.

10. $g_B = \phi g_A/2$ (benchmark), with 0 and $g_A/2$ (at minimum and maximum): For g_B, I consider two extreme values, minimal and maximal, and simply utilize the midpoint as a benchmark. However, I also report the calibration results based on the two extreme values as robustness checks. At one extreme, $B(t)$ can be considered constant. People are born the same at different times with the same level of B. However, people born later learn faster due to the accumulated "general knowledge" including the improvement of learning technologies. Such knowledge is captured by $\overline{h}(t)$, so B can be constant. Many studies, theoretical and empirical, follow this simple approach, including Lucas (1988, 2004) and Tamura (1991). At the other extreme, if $B(t)$ is viewed as non-rival technologies, e.g., classroom equipment, which are separate from the general knowledge, then $B(t)$ will grow. Notice that a part of $A(t)$ growth is the advances in non-rival technologies. However, since $A(t)$ includes other sources in productivity growth (such as an elimination of the monopoly power), it is plausible to assume that productivity growth in human capital production is slower than it is in goods production. I assume that all $A(t)$ growth is due to non-rival technologies to allow the highest growth rate for $B(t)$. To be specific, I specify the law of motion of human capital as

$$\dot{h}(t) = B\left[A(t)u(t)h(t)\right]^{\phi}\overline{h}(t)^{\theta} - (\delta_h + g_L)h(t),$$

where $B > 0$, so that $B(t) = BA(t)^{\phi}$, and hence, $g_B = \phi g_A$. That is, $A(t)$ enhances human capital input, not only in goods production but also in human capital production.[17] The benchmark that I take is the mid-point of the two extremes, $g_B = 0$ and $g_B = \phi g_A$.

year of schooling increases the labor productivity by 10%. Then, individual human capital stock can be measured by $e^{0.1S}$ where S is the years of schooling. Using Barro and Lee's (1996, 2001) data to compute this in 1960 and 2000, I have a growth accounting result that about 1/3 of per-capita output growth is explained by schooling. Since human capital externalities are not considered, this measurement, 1/3, is the minimum.

[17] Without this identifying assumption, the growth of $B(t)$ can be measured by alternative data obser-

One issue is whether the data support my assumption that u is constant on the balanced growth path. According to Barro and Lee (1996, 2001), U.S. years of schooling steadily increased since 1960. The Digest of Education Statistics, published by the U.S. Department of Education, shows that educational expenditure (including teacher salaries) increased from 4.7 percent of GDP in 1960 to 7.5 percent in 2005. As previously discussed, R&D expenditures as a fraction of GDP were stable, at least in 1988-2004, according to OECD Main Science and Technology Indicators. No data that show the time-series trend of the on-the-job training is available. Thus, at the surface, it appears that u had increased over time. However, some of the increase in schooling could be that it is treated, in part, as a normal consumption good (which is not captured by my framework).[18] Another possibility is that the formal schooling simply took over the role of on-the-job training. Alternatively, years of schooling may have increased just because the life expectancy increased. In sum, schooling expanded both in years and in expenditure, but the data on other components of human capital production are not sufficient to conclude whether u indeed increased or not.

3.2. Calibration of the Main Equations

Equation (6) implies $g_A = 0.015$. Equation (7) becomes

$$g_B = 0.007(1 - \phi - \theta), \tag{15}$$

where $g_B = \phi g_A/2$ by assumption. The equations, (8), (9), (10), (11), (12), and (13), are parameterized as

vations. For example, differences in taxes or subsidies, across economies or over periods, may identify the growth rates of $h(t)$ and of $B(t)$ separately. Also, one may compare the performances of identical students in two identical schools (with the same $B(t)$) with different qualities of teachers and peers.

[18] Bils and Klenow (2000) introduce the utility gain from schooling in their model.

$$
\begin{aligned}
&(16) \quad & 0.033 + \delta_K &= 0.21/(K/Y), \quad \text{where } \delta_K (K/Y) = 0.12 \text{ from data,} \\
&(17) \quad & 0.033 - 0.015 &= B_0 h_0^{\phi+\theta-1} 0.28^{\phi} - 0.035, \\
&(18) \quad & \rho + \sigma\,(0.033 - 0.011) &= 0.728r - \delta_K \\
&(19) \quad & &= 0.728 \times \frac{0.34}{K/Y} - \delta_K, \\
&(20) \quad & &= 1.277\phi B_0 h_0^{\phi+\theta-1} 0.28^{\phi-1} - 0.035 + 0.015 \\
&(21) \quad & &= 0.670 \times \frac{(1-0.34)(K/Y)^{0.34/(1-0.34)}}{V_0} - 0.035 + 0.015
\end{aligned}
$$

Two constants, δ_K and (K/Y), are calibrated solely from (16). Then, the equality between the right-hand sides of (18) and (19) provides a calibrated value of r. The solutions to these three unknowns are solely based on goods production and physical capital accumulation, so they are not affected by the form of the human capital production function. Then, from (17) and (20), ϕ and $B_0 h_0^{\phi+\theta-1}$ are separated. Since I have the calibrated values of ϕ and g_A, I obtain g_B from $g_B = \phi g_A/2$. Then, θ can be calibrated by (15). Finally, V_0 is solved for in (21). The calibration result is summarized in Table 1.

The parameter of most interest is θ since it tells us whether learning externalities exist. Table 1 reports $\theta = 0.44$. A sensitivity check of θ with respect to other constants is reported in Part (C) of Table 1. First, if g_A converges to zero so that the model becomes purely endogenous, then $\theta = 0.73$. Second, as g_B moves from zero (at the minimum) to 0.4 percent (at the maximum), the calibrated value of θ moves from 0.72 to 0.16, but it stays above zero. Third, if human capital depreciates only due to retirement (and there is no individual depreciation), then $\delta_h = 0.022$ and $\theta = 0.41$. On the other hand, if individual depreciation is the only source of human capital depreciation (and there is no retirement), then $\delta_h = 0.013$ and $\theta = 0.36$. Hence, calibrated values are substantially greater than zero even with alternative values of δ_h.

////////// INSERT TABLE 1 AROUND HERE //////////

The meaning of $\theta = 0.44$ is most evident from (17) and (20). Equation (17) relates the total product of human capital to per-capita output growth. Equation (20) equates the (private) marginal product of human capital to the interest rate. Alternatively, one may interpret (17) as a condition on the *social* returns to human capital and (20) as a condition on the *private* returns. For simplicity, disregard g_h for now and assume $g_A = g_Y - g_L$. Then in (17), the left-hand side becomes population growth (g_L), 1.1 percent. In addition, 3.5 percent of the human capital stock needs to be replaced due to depreciation ($\delta_h = 0.035$). This situation is as if there were 100 workers, but 3.5 workers left, and 4.6 new workers arrived. In other words, 4.6 new workers should be "produced" by the original 100 workers. Since the after-tax-and-subsidy net interest rate is 4.6 percent, equation (20) implies that the marginal product of human capital in human capital production is, roughly, 5 percent (or 5.2 percent if δ_h, g_A and the constant 1.277 are properly considered). Thus, without learning externalities, it would take approximately 92 workers to produce the required 4.6 workers. This number is high compared to 28 workers, which the observation $u = 0.28$ suggests.

The gap in this example is filled with learning externalities. In (20), the net after-tax-and-subsidy interest rate of 4.6 percent fixes the marginal product of human capital. Thus, the issue is how once can increase the total human capital production while keeping this marginal product constant. In the model, workers benefit from each other in producing new workers. When $\theta = 0$ (no learning externalities), about 92 workers could produce 4.6 workers. However, when $\theta = 0.44$ and $\phi = 0.28$, then the contribution from direct input is only 39 percent ($= 0.28/(0.44 + 0.28)$), so only 36 workers ($= 0.39 \times 92$) are required, which is closer to 28 workers ($u = 0.28$). Recall that I disregarded g_h in this example. Thus, one can interpret that the difference between these two numbers, 28 and 36, are dedicated to increase the per-capita human capital. To summarize, the key variables that are informative with respect to learning externalities are (i) u, (ii) $r^* - \delta_K$, and (iii) $g_L + \delta_h$. Learning externalities are large because u and $r - \delta_K$ are low, which restricts the marginal product of human capital, while $g_L + \delta_h$ is high.

Some micro-level studies consider an individual-level human capital production function, $d\widetilde{h}_t/dt = \widetilde{B}(\widetilde{u}_t\widetilde{h}_t)^{\widetilde{\phi}} - \widetilde{\delta}_h\widetilde{h}_t$. (Tildes emphasize variables or parameters on the individual level.) The estimated value of $\widetilde{\phi}$ is usually between 0.5 and 1 (see Browning, Hansen and Heckman (1999) for details). This is higher than my calibration on ϕ. Possible reasons for this difference are as follows. First, I explicitly consider learning externalities in the calibration. The effect of such externalities may be absorbed in $\widetilde{\phi}$ in micro-level studies. Second, δ_h in this paper includes the loss of per-capita human capital due to retirements while $\widetilde{\delta}_h$ does not. Third, u_t in this paper includes on-the-job training while $\widetilde{u}_t$ usually does not. I discuss the empirical labor literature further in Section 4.

3.3. Private Value of Human Capital

To understand the role of human capital production more clearly, I obtain the private value of human capital stock and the private value of human capital production. First, to consider the value of human capital stock, notice that from (6) and $g_V = g_A$, the value of human capital stock relative to goods production, or $V(t)h(t)L(t)/Y(t)$, is constant on the balanced growth path. Since I assumed $A(0) = 1$ without loss of generality,

$$\frac{V(t)h(t)L(t)}{Y(t)} = \frac{V_0}{(K/Y)^{\alpha/(1-\alpha)}(1-u)} = 9.25. \tag{22}$$

That is, the human capital stock is worth 9.25 times goods production, or equivalently, 8.64 times GDP. Using the human capital production function specified in (5), I can also compute the value of economy-wide human capital production, relative to goods production, as

$$\frac{V(t)B(t)\left[u(t)h(t)\right]^{\phi}\overline{h}(t)^{\theta}L(t)}{Y(t)} = \frac{V(t)h(t)L(t)}{Y(t)} \times B_0 h_0^{\phi+\theta-1} u^{\phi} = 0.49. \tag{23}$$

That is, the human capital production is worth 49 percent of goods production, or equivalently, 46 percent of GDP. Since the economy-wide human capital production function can be written as $B\left[A(t)u(t)h(t)L(t)\right]^{\phi} L(t)^{1-\phi-\theta}\left[\overline{h}(t)L(t)\right]^{\theta}$, I interpret that fractions ϕ and $1-\phi-\theta$ of human capital production are contributions from human capital and raw labor, while the fraction θ is from learning externalities. Then, the income of human capital and raw labor, in human capital production, is worth 26 percent of GDP, while the gain from learning externalities is 20 percent of GDP.

3.4. Social Return on and Social Value of Human Capital

In this subsection, I estimate the social rate of return on human capital and the social value of human capital. To develop a formula for the social return, recall that the individual-level human capital production function is $B(t)\,[u(t)h(t)]^{\phi}\,\overline{h}(t)^{\theta}$. The private marginal product of human capital (before taxes and subsidies) is obtained by differentiating this function with respect to $u(t)h(t)$. That is, it is $B(t)\phi[u(t)h(t)]^{\phi-1}\overline{h}(t)^{\theta}$, which is included in the right-hand side of (12) as $\phi B_0 h_0^{\phi+\theta-1}u^{\phi-1}$ in a period-0 term. The difference between social and private returns comes from learning externalities. By differentiating the human capital production function with respect to $\overline{h}(t)$, the marginal product of human capital (before taxes and subsidies) *for providing learning externalities* is obtained as $B(t)\,[u(t)h(t)]^{\phi}\,\theta\overline{h}(t)^{\theta-1}$. Hence,

$$\text{(social return)} = \text{(private return)} + \theta B_0 h_0^{\phi+\theta-1}u^{\phi}. \tag{24}$$

From the calibration results, the private return on human capital, after depreciation (δ_h) and exogenous productivity growth (g_A), is equal to $r^* - \delta_K = 0.046$, which is direct from (12). Before depreciation and exogenous productivity growth, this return is $r^* - \delta_K + \delta_h - g_A$ =0.066. This is the private marginal product of human capital which is the first term of the right-hand side of (12). The second term of the right-hand side of (24), which is the difference between social and private returns, is 2.4 percent. Hence, the social return on human capital (or equivalently, social marginal product of human capital) becomes 9.0 percent. These results are summarized in Table 2.

////////// INSERT TABLE 2 AROUND HERE //////////

To formally derive (24), one can solve the consumer's problem of (3), (4) and (5), taking $\overline{h}(t)$ as *endogenous* (i.e., imposing $\overline{h}(t) = h(t)$ in advance). Then, (12) and (13) are replaced by

$$\rho+\sigma\left(g_C-g_L\right)=\left[\frac{1-\alpha\tau_K-(1-\alpha)\tau_L-\frac{\alpha su}{1-u}}{1-\alpha\tau_K-(1-\alpha)\tau_L-\frac{\alpha su}{1-u}-s}\right]\phi B_0h_0^{\phi+\theta-1}u^{\phi-1}+\theta B_0h_0^{\phi+\theta-1}u^{\phi}-\delta_h+g_A$$
$$=\left[1-\alpha\tau_K-(1-\alpha)\tau_L-\frac{\alpha su}{1-u}\right]\frac{(1-\alpha)(K/Y)^{\alpha/(1-\alpha)}}{V_0}$$
$$+\theta B_0h_0^{\phi+\theta-1}u^{\phi}-\delta_h+g_A$$

The right-hand sides of these two equations have an additional term denoting the difference between private and social returns, as in (24). Then, (14) is similarly replaced by

$$(r^*-\delta_K)V_0=\left[1-\alpha\tau_K-(1-\alpha)\tau_L-\frac{\alpha su}{1-u}\right](1-\alpha)(K/Y)^{\alpha/(1-\alpha)}$$
$$+\left(\theta B_0h_0^{\phi+\theta-1}u^{\phi}-\delta_h+g_A\right)V_0$$

This equation provides a further asset-pricing implication of the model. One unit of human capital with initial value V_0 in period 0 would deliver the first-term of the right-hand side (i.e., after-tax-and-subsidy marginal product) as a dividend and have a value increase by the second-term of the right-hand side through (i) learning externalities, (ii) depreciation, and (iii) exogenous productivity growth. More specifically, after one period, the asset delivers $\theta B_0h_0^{\phi+\theta-1}u^{\phi}$ units of the same asset (i.e., human capital), as the gains due to learning externalities. At the same time, by depreciation, a fraction δ_h of the asset disappears, but through an increase of the unit value, each unit is now worth $e^{g_A}V_0$ due to the productivity growth.

Solving for V_0 in this equation provides the *social* value of one unit of human capital, including the value of learning externalities that the human capital generates in all future periods. From the calibration results, V_0 =17.57. This is higher than its private counterpart, 11.36, reported in Table 1. Revisiting (22) and (23) with this social value suggests that the social value of economy-wide human capital stock is 13.37 times GDP and that the social value of human capital production is worth 71 percent of GDP.

Table 3 summarizes the findings so far. It consists of three versions of national accounts. The first is the one currently adopted, without considering foregone human capital income due to human capital production or the gains from learning externalities. I normalize GDP to 100. Since human capital income in educational services and scientific R&D services is 7 percent of GDP (as discussed in Subsection 3.1), goods production is 93 and human capital production is 7. The second account includes foregone human capital income due to human capital production and the gains from learning externalities.[19] The human capital stock is measured using the private value, or V_0 =11.36, so that the account is only about the private compensation that human capital receives. On the other hand, the third account updates the second by using the social value of human capital, V_0 =17.57. Here, human capital is valued including the learning externalities that it will generate in all future periods.

////////// INSERT TABLE 3 AROUND HERE //////////

3.5. Physical Goods as an Input to Human Capital Production

I now consider a model with physical goods as an input in human capital production. That is, now let

$$
\begin{aligned}
Y &= [(1-v)K]^{\alpha}[A(1-u)hL]^{1-\alpha}, \\
\dot{K} &= I - \delta_K K, \\
\dot{h} &= B(vK/L)^{\psi}(uh)^{\phi}\overline{h}^{\theta} - (\delta_h + g_L)h, \\
Y &= I + C + \tau_C C + \tau_K(1-v)rK + \tau_L wA(1-u)hL - s(rvK + wAuhL),
\end{aligned}
$$

for $\psi > 0$ and $\psi + \phi + \theta < 1$, where $r \equiv \alpha[(1-v)K]^{\alpha-1}[A(1-u)hL]^{1-\alpha}$, $w \equiv (1-\alpha)[(1-v)K]^{\alpha}[A(1-u)hL]^{-\alpha}$, $\dot{A} = g_A A$, $\dot{B} = g_B B$ and $\dot{L}_t = g_L L_t$. Here, v is a fraction of physical capital stock devoted to human capital production, which is the physical capital counterpart to u.

[19] Jorgenson and Fraumeni (1989) propose a similar account, but under an implicit assumption that there are no human capital externalities.

I assume that all previous equilibrium conditions hold. However, there is an additional condition equating the marginal products of physical capital in two sectors, goods production and human capital production. In calibration, there are two additional unknown constants, v (constant on the balanced growth path) and ψ, so I need an additional observation from the data. In Klenow and Rodríguez-Clare (1997), the share of physical capital as an input of human capital production values about 10 percent out of all direct inputs, implying $\psi/(\psi+\phi) = 0.1$. Recall that to calibrate ϕ, I use the conditions on total product and marginal product of human capital, so introducing ψ does not substantially affect ϕ. Then, ϕ continues to be around 0.28, leaving ψ to be around 0.03. Since ψ is small, the calibrated value of θ is only slightly affected. Therefore, an introduction of physical capital in human capital production does not substantially affect the results.

4. Empirical Labor Literature

The empirical labor literature reports some useful observations that are relevant to the calibration in this paper. In Subsection 4.1, I discuss the empirical findings on the (in)equality between the returns on human capital and physical capital. In Subsection 4.2, I discuss how income convergence across the U.S. states and the evolution of teachers' wages relative to the per-capita GDP can be understood in the framework introduced here. In Subsection 4.3, I consider the role of in-home training before formal schooling.

4.1. (In)Equality between the Returns on Human Capital and Physical Capital

The benchmark calibration equates the returns on human capital and physical capital, after controlling for taxes, subsidies, productivity growth and depreciation. This equality may not hold for several reasons. First, there can be a borrowing constraint for younger generations, so the return on education may be higher. Second, human capital and physical capital may have different return volatilities, so the expected returns in an equilibrium may be different. Third, utility or disutility may result from education, as in Bils and Klenow (2000), so pecuniary returns may be different.

Based on micro-level estimations, Heckman, Lochner and Todd (2008) suggest that the return on human capital in human capital production is about twice as high as the return on physical capital. To see how this affects the calibration, I start by replacing equations (10) through (13) by

$$\begin{aligned}
&\left[1-\alpha\tau_K-(1-\alpha)\tau_L+\frac{(1-\alpha)su}{1-u}\right]r-\delta_K \\
&=\left[1-\alpha\tau_K-(1-\alpha)\tau_L+\frac{(1-\alpha)su}{1-u}\right]\frac{\alpha}{K/Y}-\delta_K \\
&=\frac{1}{2}\times\left(\frac{1-\alpha\tau_K-(1-\alpha)\tau_L-\frac{\alpha su}{1-u}}{1-\alpha\tau_K-(1-\alpha)\tau_L-\frac{\alpha su}{1-u}-s}\phi B_0 h_0^{\phi+\theta-1}u^{\phi-1}-\delta_h+g_A\right) \\
&=\left[1-\alpha\tau_K-(1-\alpha)\tau_L-\frac{\alpha su}{1-u}\right]\frac{(1-\alpha)(K/Y)^{\alpha/(1-\alpha)}}{V_0}-\delta_h+g_A
\end{aligned}$$

Notice that the return on human capital in *goods* production is still assumed to be the same as the return on physical capital. This is reasonable because the borrowing constraint on education, the return volatility from education, and the utility or disutility from education are relevant to human capital production and not relevant to goods production. The calibration results based on this equation system are summarized in Part (A) of Table 4.

////////// INSERT TABLE 4 AROUND HERE //////////

As I assume the return on human capital in human capital production is 1.5 or 2 times as high as the returns on physical capital, the calibrated value of θ decreases from 0.44 (at the benchmark) to 0.25 or 0.05. The difference between social and private returns on human capital also decreases from 2.3 percent (at the benchmark) to 1.3 percent or 0.3 percent. Hence, learning externalities are calibrated to be negligible if Heckman, Lochner and Todd's (2008) result is adopted. Notice that it does not change a policy implication that education (or human capital production) is under-invested when there is no government intervention. This is because the internal rate of return on education is still higher than the interest rate, whether there exist learning externalities or not, as argued in Heckman, Lochner and Todd (2008, pp. 1-2).

Although this exercise is informative, it does not imply that my benchmark calibration, which suggests that learning externalities are sizable, is flawed. First, the results discussed in Section 3 are based on a conservative assumption that g_A is as high as 2/3 of per-capita output growth. This assumption implies that education, R&D, on-the-job training, and human capital externalities explain only 1/3 of per-capita output growth. On the other hand, Heckman, Lochner and Todd (2008) assume $g_A = 0$. To understand their result in the context of my analysis, in their equation (5), the variable r_l should be replaced by $r_l + g_A$, as in this paper's equations (12) and (13). In other words, their estimate of r_l would be lowered by g_A if exogenous productivity growth is introduced. This implies that the return on human capital in human capital production is less than twice as high as the return on physical capital. If I assume $g_A = 0$ in my calibration, then the result still suggests sizable learning externalities, as reported in Part (B) of Table 4.

Second, Heckman, Lochner and Todd (2008) assume that education is the only source of human capital production, while my calibration also considers R&D and on-the-job training. Table 5 reports average earnings of high school graduates and college graduates relative to national average earnings. At age 30, college graduates make about 30 percent more earnings than high school graduates. This gap increases for the next 20 years, so at age 50, college graduates make about 70 percent more. An interpretation is that college graduates are more actively engaged in the on-the-job training. If one does not include the on-the-job training in the analysis, the income difference must come fully from education. This gives an upward bias to the return on education. Hence, if on-the-job training is considered, the return on education can be substantially lower than Heckman, Lochner and Todd's (2008) estimate.

////////// INSERT TABLE 5 AROUND HERE //////////

4.2. Output Convergence across the U.S. States

In this subsection, I discuss Tamura (2001) to understand my calibration results more fundamentally. Tamura (2001) estimates the human capital production function with primary and secondary education. The inputs include teacher quality, teacher quantity (or class size), enrollment rate, and school length. The estimated production function mostly explains the output growth of 1880s-1990s in U.S. states.

I raise two questions. First, do Tamura's (2001) results imply that learning externalities are not important? As Tamura (2001) reports, the state-level per-capita outputs converged. A common pool of the *knowledge*, or equivalently, learning externalities, can be a contributor to this convergence. However, Tamura (2001) explains this by the evolution of the four inputs. Especially, a common pool of *teachers* in hiring across the states is an important contributor. Thus, even without learning externalities, the convergence can be explained. The second question is how one can understand a decrease in a teacher's relative wage, observed in data, in my calibration framework. As Tamura (2001) reports, the average teacher quality, measured by a teacher's wage relative to the per-capita GDP, decreased from 2.35 in 1960 to 1.76 in 1990.

My answer to the first question is no. Learning externalities in my framework can be applied to state-level or country-level economies. To be specific, low-income economies can learn knowledge that has been discovered in high-income economies through various types of interactions (such as communications and the trade of goods and services), often without compensating for the knowledge transfer. Tamura (2001) suggests that a common pool of teachers is important. However, the country-level economies, which rarely trade teachers in primary and secondary schools, also converged. Figure 1 shows that the convergence is observed not only for U.S. states but also for the West European countries in 1963-2005.[20] This implies that learning externalities can be still important.

[20] The U.S. state-level per-capita GDPs are obtained from the U.S. Bureau of Economic Analysis and the U.S. Census Bureau. The data are available from 1963. The per-capita GDPs in West European countries are obtained from the World Development Indicators 2009, published by the World Bank. The price levels are adjusted with consumer-price-index inflation data from the U.S. Bureau of Labor

////////// INSERT FIGURE 1 AROUND HERE //////////

It is instructive to decompose the output convergence of the U.S. states into the contribution from learning externalities and the contribution from a common pool of teachers. To do so, I assume a new law of motion of human capital which is different from the one used in my benchmark calibration. This is because the issue here is a cross section of economies while the benchmark calibration focused on a single economy. Furthermore, the state-level data on $u(t)$ are not easily available, and hence, an application of my original human capital production function is difficult. I assume that economy i's per-capita human capital, $h_i(t)$, evolves as

$$\dot{h}_i(t) = B[h_i(t)]^{1-\zeta}[\overline{h}(t)]^{\zeta},$$

where $\overline{h}(t)$ is the per-capita human capital of an average economy. Here, $0 < \zeta < 1$ can be interpreted as a speed of convergence, including both learning externalities and the effect of a common pool of teachers.[21] As in Tamura (2001), I assume that there is no exogenous productivity growth in goods production ($g_A = 0$). Then, as in my benchmark calibration, B is constant. Also, as in Tamura, I assume that the per-capita output is proportional to the per-capita human capital.

The above law of motion implies $\log[\dot{h}_i(t)/h_i(t)] = \log B + \zeta \log[\overline{h}(t)/h_i(t)]$. Given a set of state-level or country-level economies, I can apply the method of ordinary least squares (OLS) to estimate ζ, given the growth rate of per-capita output and the initial level of per-capita output relative to an average economy. The data for the U.S. states in 1963-2005 provides 0.48 as an estimate of ζ. The data for the West European countries provides 0.33. Therefore, the convergence is faster in the U.S., which can be interpreted as the effect of a common pool of teachers (and other domestic transactions). That is, the difference between these two speeds, which is as large as about 1/3 of the U.S. speed, is a contribution from a common pool of teachers (and other domestic transactions). The remaining 2/3 is the effect of a common pool of knowledge, which the West European countries also benefit from.

Statistics.

[21] Lucas (2009) provides a similar interpretation in a related set-up.

Although the law of motion of human capital is different from the one in my benchmark, an application of this decomposition is instructive. Now I assign only 2/3 of the originally calibrated value to θ in my benchmark framework. With this new parameter value, I can obtain the social return on human capital by the same formula. Table 6 shows how the result is affected. Depending on the assumption on g_A, the social return on human capital is 1.6−3.3 percent points higher than the private return. The result still suggests that sizable learning externalities exist.

////////// INSERT TABLE 6 AROUND HERE //////////

Now I consider the original calibration framework and address the second question. How can one understand a decrease in a teacher's relative wage in my calibration framework? A possibility is that the economy moved from one balanced growth path to another, through structural changes such as tax and subsidy reforms. As discussed in the calibration of s and u in Subsection 3.1, each unit of human capital in human capital production earns $\phi B_0 h_0^{\phi+\theta-1} u^{\phi-1} V_0 + sw$, at period 0 in which $A(0) = 1$. One can easily see that the wage per worker relative to per-capita GDP is $\left[1 - \alpha\tau_K - (1-\alpha)\tau_L - \frac{\alpha su}{1-u}\right] wh_0/(Y(0)/L(0))$. Since $w(1-u)h_0 L(0) = (1-\alpha)Y(0)$, this ratio is equivalent to

$$\left[1 - \alpha\tau_K - (1-\alpha)\tau_L - \frac{\alpha su}{1-u}\right] \frac{1-\alpha}{1-u}.$$

Since a teacher's wage relative to the per-capita output was 2.35 and 2.30 in 1960 and 1970 and then dropped to 1.76 in both 1980 and 1990 (Tamura 2001, Table 1), it is plausible to assume that the economy was on separate balanced growth paths in 1960s and 1980s, while it was in transition in 1970s. I calibrate for 1960s and 1980s separately, based on the following:

1. $\tau_K = 0.423$ (1960s) $\longrightarrow$ 0.421 (1980s): See Subsection 3.1 for the data source.[22]

2. $\tau_L = 0.201$ (1960s) $\longrightarrow$ 0.285 (1980s): See Subsection 3.1 for the data source.

[22] Hence, I do not observe a sizable change in a tax rate on physical capital income. During 1980s, the corporate income tax decreased, but the model does not have an explicit corporate sector.

3. Subsidies to human capital production: There is no evidence of a significant change in the subsidies to human capital production as a fraction of GDP. A fraction of public educational expenditure out of total educational expenditure is similar as 75 percent in 1960s and 74 percent in 1980s. (See Subsection 3.1 for the data source.)

4. Human capital income in human capital production: Educational expenditure increased from 5.6 percent of GDP (1960s) to 6.3 percent (1980s). As discussed in Subsection 3.1, there is no evidence of a change in other components of human capital production. Hence, I assume that only educational expenditure increased. This change in human capital income in human capital production, as well as the change in τ_L, affects the calibrated values of s and u. The value of s changes from 16.6 percent (1960s) to 13.2 percent (1980s). The value of u changes from 0.27 (1960s) to 0.29 (1980s).

The calibrations on the two balanced growth paths provide similar estimates of θ, 0.45 (1960s) and 0.44 (1980s). In Table 7, a teacher's wage relative to the per-capita GDP in data is reported. It also reports the model's prediction, assuming that a teacher is an average worker in human capital production. The model, reflecting the changes in taxes and subsidies, explains a 4 percent decrease in a teacher's relative wage. This is only 1/5 of the observed decrease.

////////// INSERT TABLE 7 AROUND HERE //////////

There is another possibility. A large part of output growth comes from exogenous productivity growth and improvements in tertiary education, R&D and on-the-job training. Of course, the curricula in the primary and secondary education improved, but their essential parts remained the same. The human capital of a teacher may have grown only slowly while the human capital of an average worker and exogenous productivity in goods production grew faster. To reflect this possibility, I assume that each teacher's human capital remains the same between 1960s and 1980s, while an average worker's human capital still grows at g_h. As Table 7 reports, this can explain almost 3/4 (= 17/24) of a decrease in a teacher's relative wage.

Furthermore, a decision on the teacher quantity (or class size) can be optimally made to maximize the returns to schooling, by students (and their parents) and by the government, within certain resource constraints. Given this decision over time, a teacher's marginal product may decrease because a teacher has become responsible for fewer students (even though each student's return to schooling may have increased). To fully analyze this problem, one needs to introduce a theory of optimal class size and then interpret the result in an equilibrium of worker allocations between goods production and human capital production. While this is beyond the model's capacity, it can be important because a teacher's wage *per student* relative to per-capita GDP, obtained by the relative wage divided by class size using Tamura's (2001) Table 1, had been relatively stable, at 0.093 in 1960, 0.101 in 1970, 0.096 in 1980 and 0.104 in 1990. Another point that can help one to understand the decrease in a teacher's relative wage is an observation that the teaching profession has "tipped" from being predominantly male to predominantly female. Historically, female-dominated professions have earned less, on average, compared with male-dominated professions.

The decrease in a teacher's relative wage plays only a limited role in providing a more accurate estimate of learning externalities. First, as I discussed, a majority of this decrease can be understood in the benchmark calibration. Second, the human capital production in this paper considers not only students and teachers in primary and secondary education, but also those in tertiary education and the labor force engaged in R&D and on-the-job training. As Subsection 3.1 discusses, human capital income in educational services plus the foregone income due to schooling is 6.1 percent of GDP. This is only about 1/3 of all human capital income in human capital production, 16.3 percent of GDP. Furthermore, this includes the tertiary education which is not included in Tamura's (2001) production function.

4.3. In-Home Training

Manuelli and Seshadri's (2007) cross-country analysis implies that schooling and on-the-job training are the dominant contributors to human capital accumulation, while investments in early childhood play a minor role. While the benchmark calibration excludes in-home training before formal schooling out of human capital production, how will including it affect the result?

I consider a measurement on the foregone income of parents due to the time directly engaged in an activity with their children of ages 1-7.[23] According to the Population Estimates Program of the U.S. Census Bureau, 6.8 percent of total population is roughly in this age group in 2000. Based on Sandberg and Hofferth (2005), a child's hours directly engaged with either parent in an activity are, on average, 25.5 hours per week in 1997.[24] According to the Current Employment Statistics of the U.S. Bureau of Labor Statistics, total employed workers are 46.7 percent of total population in 2000, and average working hours are 34.3 hours per week. Then, I can compare the total man-hours of the economy spent in in-home training (6.8×25.5) and working (46.7×34.3). These two numbers imply that if all hours engaged in an activity with children were to be transferred to working, then additional earnings would be 10.8 percent of the current human capital income. This is worth 6.9 percent of GDP. Adding this to human capital income in educational services and R&D and foregone income due to schooling and on-the-job training gives 23.2 percent of GDP. Assuming there is no government subsidy to in-home training, this new number gives 0.23 as a calibrated value of θ. The social return on human capital becomes 7.9 percent, which is 1.2 percent points higher than the private return, 6.6 percent. Hence, learning externalities are still sizable even after adding in-home training to human capital production activities.

[23] For example, a parent's reading with children enhances their verbal and cognitive skills. Leibowitz (2003) reviews the literature on the importance of in-home training in human capital accumulation.

[24] Sandberg and Hofferth (2005) report that these hours are similar whether mothers work in the labor market or not. Observations for the entire span of 1960-2005 are not available.

5. Concluding Remarks

This paper estimates learning externalities by calibrating a two-sector growth model using U.S. data. The results suggest that sizable learning externalities exist, even in a conservative set-up. The social rate of return on human capital is 2.4 percent points higher than the private rate of return of 6.6 percent. In many growth accounting exercises, human capital stock is measured by a count of equivalent college graduates. However, if human capital externalities are important, as this paper suggests, then the human capital holding of an average college graduate will increase over time as more knowledge becomes available. A more accurate measurement of human capital would require a better understanding of human capital externalities.

The calibration in this paper estimates learning externalities only. Static externalities, which may appear in goods production, can also be an important source of human capital externalities. Including static externalities does not affect the estimates of learning externalities. This is because the model has exogenous productivity which can be alternatively interpreted as static externalities. Since micro-level estimations on static externalities vary from a negligible level (as in Acemoglu and Angrist (2000)) to a level as high as the private return on human capital (as in Liu (2007)), the social return on human capital including both forms of externalities will be between 9.0 percent and 15.6 percent.

Introducing an overlapping generations model with data observations on age-earnings profiles would be informative for a future study in order to clearly distinguish g_h from g_A. Rangazas (2005) and Manuelli and Seshadri (2007) are a good starting point in this direction. Future estimates of learning externalities would be higher than my calibrations because I assumed the maximum level of g_A generally considered in the literature.

While this paper considered a semi-endogenous growth model, it would be also useful to engage in a similar exercise in an exogenous growth model in which output levels, not growth rates, are endogenously determined.

Finally, while this paper focuses on the United States as a closed economy, one can extend the model to an international set-up with international externalities. A calibration of transitional paths can provide a better understanding of catch-up growth episodes.[25]

* School of Economic Sciences, Washington State University, U.S.A.

Appendix

A.1. Solving for an Equilibrium on the Balanced Growth Path

I rewrite the consumer's problem in an intensive form. Define $c(t) \equiv C(t)/A(t)^{\frac{g_B}{g_A}\frac{1}{1-\phi-\theta}+1}L(t)$, $k(t) \equiv K(t)/A(t)^{\frac{g_B}{g_A}\frac{1}{1-\phi-\theta}+1}L(t)$, $\tilde{h}(t) = h(t)/A(t)^{\frac{g_B}{g_A}\frac{1}{1-\phi-\theta}}$, and $\tilde{\overline{h}}(t) = \overline{h}(t)/A(t)^{\frac{g_B}{g_A}\frac{1}{1-\phi-\theta}}$. Without time indicators, the problem of the representative consumer, described as (3), (4) and (5), is written as

$$\max_{\{c,u\}_{t=0}^{\infty}} \int_0^{\infty} \exp(-\eta t)\frac{c^{1-\sigma}}{1-\sigma}dt, \tag{25}$$

subject to

$$\dot{k} = \left[d + \frac{(1-\alpha)su}{1-u}\right] k^{\alpha}[(1-u)\tilde{h}]^{1-\alpha} - (1+\tau_C)c - ek, \tag{26}$$

$$\dot{\tilde{h}} = \bar{B}u^{\phi}\tilde{h}^{\phi}\tilde{\overline{h}}^{\theta} - f\tilde{h}, \tag{27}$$

given $k(0)$, $\tilde{h}(0)$, and $\{\tilde{\overline{h}}(t)\}_{t=0}^{\infty}$, where

$$\begin{aligned}
\eta &\equiv \rho - g_A\left(\frac{g_B}{g_A}\frac{1}{1-\phi-\theta} + 1\right)(1-\sigma) - g_L, \\
d &\equiv 1 - \alpha\tau_K - (1-\alpha)\tau_L, \\
e &\equiv \delta_K + \left(\frac{g_B}{g_A}\frac{1}{1-\phi-\theta} + 1\right) g_A + g_L, \\
f &\equiv \delta_h + g_L + \frac{g_B}{1-\phi-\theta}, \\
\bar{B} &\equiv B(0)/A(0)^{\frac{g_B}{g_A}}.
\end{aligned}$$

With the calibrated values reported in the text, $\eta > 0$ is satisfied so that a usual Hamiltonian technique can be applied. By definition, $d > 0$, $e > 0$ and $f > 0$. The Hamiltonian

[25] For the measurements of international R&D externalities, see Eaton and Kortum (1999) and Klenow and Rodríguez-Clare (2005).

function, with c and u as control variables, k and $\tilde{h}$ as state variables, and λ and μ as corresponding co-state variables, is

$$\mathcal{H}(c, u, k, \tilde{h}, \lambda, \mu) = \frac{c^{1-\sigma}}{1-\sigma} + \lambda \left(\left[d + \frac{(1-\alpha)su}{1-u} \right] k^{\alpha} [(1-u)\tilde{h}]^{1-\alpha} - (1+\tau_C)c - ek \right) + \mu \left(\bar{B} u^{\phi} \tilde{h}^{\phi} \tilde{\bar{h}}^{\theta} - f\tilde{h} \right).$$

The first-order conditions are

$$c^{-\sigma} = (1+\tau_C)\lambda, \tag{28}$$

$$0 = \lambda \left[d - s - \frac{\alpha su}{1-u} \right] (1-\alpha) k^{\alpha} (1-u)^{-\alpha} \tilde{h}^{1-\alpha} - \mu \bar{B} \phi u^{\phi-1} \tilde{h}^{\phi} \tilde{\bar{h}}^{\theta}, \tag{29}$$

$$\eta\lambda - \dot{\lambda} = \lambda \left(\left[d + \frac{(1-\alpha)su}{1-u} \right] \alpha k^{\alpha-1} (1-u)^{1-\alpha} \tilde{h}^{1-\alpha} - e \right), \tag{30}$$

$$\eta\mu - \dot{\mu} = \lambda \left[d + \frac{(1-\alpha)su}{1-u} \right] k^{\alpha} (1-u)^{1-\alpha} (1-\alpha) \tilde{h}^{-\alpha} + \mu \left(B u^{\phi} \phi \tilde{h}^{\phi-1} \tilde{\bar{h}}^{\theta} - f \right), \tag{31}$$

with two transversality conditions. A system of six equations, (26) through (31), are the solutions to the maximization problem. It becomes the equilibrium conditions once $\tilde{\bar{h}} = \tilde{h}$ is imposed.

Now I look for a balanced growth path in which (i) all variables grow at constant rates (or stay at constant levels) and (ii) u is constant. Then, (27) becomes $g_{\tilde{h}} = \bar{B} u^{\phi} \tilde{h}^{\phi+\theta-1} - f$, where g_X denotes a growth rate of a variable X on the balanced growth path. Since $\phi + \theta < 1$, I have $g_{\tilde{h}} = 0$. By definition of $\tilde{h}(t)$, this implies that

$$g_B + (\phi + \theta - 1) g_h = 0. \tag{32}$$

Dividing both sides of (30) by λ, I have $g_k = 0$. Similarly, (26) gives $g_c = 0$. From (28) and (31), I also have $g_\lambda = g_\mu = 0$.

Rearranging, I can eliminate λ and μ and rewrite it as a system of the following four equations:

$$0 = \left[d + \frac{(1-\alpha)su}{1-u}\right] k^{\alpha}(1-u)^{1-\alpha}\tilde{h}^{1-\alpha} - (1+\tau_C)c - ek, \tag{33}$$

$$0 = \bar{B}u^{\phi}\tilde{h}^{\phi+\theta-1} - f, \tag{34}$$

$$\eta = \left[d + \frac{(1-\alpha)su}{1-u}\right] \alpha k^{\alpha-1}(1-u)^{1-\alpha}\tilde{h}^{1-\alpha} - e, \tag{35}$$

$$\eta = \bar{B}\phi u^{\phi-1}\tilde{h}^{\phi+\theta-1}\frac{d - \frac{\alpha su}{1-u}}{d - \frac{\alpha su}{1-u} - s} - f. \tag{36}$$

Here, (33), (34) and (35) are from (26), (27) and (30), respectively. And (36) is obtained by dividing both sides of (31) by μ and eliminating λ/μ using (29).

Proposition 2 *The solution for the consumer's problem (25) through (27), $c > 0$, $k > 0$, $\tilde{h} > 0$ and $0 < u < 1$, on the balanced growth path, exists and is unique if and only if*

$$f(1-\phi) + \eta > 0, \tag{37}$$

$$e(1-\alpha) + \eta > 0. \tag{38}$$

Proof I begin by showing that there is a unique solution for $0 < u < 1$. (34) and (36) imply

$$\frac{d(1-u) - \alpha su}{(d-s)(1-u) - \alpha su} = \frac{f+\eta}{f\phi}u.$$

The left-hand side is continuous and decreasing from $d/(d-s)$ to 1 as u increases from 0 to 1. (Recall that $d > 0$ and $s \geq 0$.) The right-hand side is continuous and strictly increasing from 0 to $(f+\eta)/f\phi$. Hence, u has a unique solution in $(0,1)$ if and only if $(f+\eta)/f\phi > 1$. Since $f > 0$ and $\phi > 0$, this is equivalent to (37). Then, (34) gives $\tilde{h} = \left(f/Bu^{\phi}\right)^{\frac{1}{\phi+\theta-1}} > 0$. Then, (35) gives

$$k = \left(\frac{e+\eta}{\left[d + \frac{(1-\alpha)su}{1-u}\right]\alpha(1-u)^{1-\alpha}}\right)^{\frac{1}{\alpha-1}}\tilde{h}, \tag{39}$$

which is strictly positive if and only if $e + \eta > 0$. But the solution for $c > 0$ given by (33) provides a stronger condition. That is, $c > 0$ if and only if

$$\left[d + \frac{(1-\alpha)su}{1-u}\right]\left[(1-u)\frac{\tilde{h}}{k}\right]^{1-\alpha} > e.$$

Rearranging using (39), I have (38). Q.E.D.

The two conditions, (37) and (38), turn out to hold for the calibrated parameter values. Now I recover the original notations. From $c \equiv C/A^{\frac{g_B}{g_A}\frac{1}{1-\phi-\theta}+1}L$, $g_c = 0$ and (32), I have $g_C = g_A + g_h + g_L$. A similar treatment for k and the use of goods production function give

$$g_Y = g_C = g_K = g_A + g_L + g_h.$$

Instead of (33) and (34), it is convenient to write their original versions, $\dot{K} = I - \delta_K K$ and (5), as

$$\begin{aligned} g_K + \delta_K &= i/(K/Y), \\ g_C - g_A &= B_0 h_0^{\phi+\theta-1} u^{\phi} - \delta_h, \end{aligned}$$

where $i \equiv (Y - C)/Y$ is the investment-output ratio and B_0 and h_0 are the values of B and h at period 0. Since K and Y grow at the same rate, I treat (K/Y) as a constant. The remaining two equations, (35) and (36) can be written as

$$\begin{aligned} \rho + \sigma\left(g_C - g_L\right) &= \left[1 - \alpha\tau_K - (1-\alpha)\tau_L + \frac{(1-\alpha)su}{1-u}\right] r - \delta_K \\ &= \left[1 - \alpha\tau_K - (1-\alpha)\tau_L + \frac{(1-\alpha)su}{1-u}\right] \frac{\alpha}{K/Y} - \delta_K \\ &= \frac{1 - \alpha\tau_K - (1-\alpha)\tau_L - \frac{\alpha su}{1-u}}{1 - \alpha\tau_K - (1-\alpha)\tau_L - \frac{\alpha su}{1-u} - s} B_0 \phi h_0^{\phi+\theta-1} u^{\phi-1} - \delta_h + g_A \end{aligned}$$

I can derive another useful equation from the first-order conditions. One can write (29) on the balanced growth path as

$$\left[1 - \alpha\tau_K - (1-\alpha)\tau_L - \frac{\alpha su}{1-u}\right] Aw = A\frac{\mu}{\lambda}\underbrace{\frac{B(0)}{A(0)^{\frac{g_B}{g_A}}} A^{\frac{g_B}{g_A}} \phi h^{\phi+\theta-1} u^{\phi-1}}_{\text{(a)}} + sAw, \tag{40}$$

where $w = (1-\alpha)K^{\alpha}[A(1-u)hL]^{-\alpha}$ is constant. Recall that Aw is the before-tax-and-subsidy rental rate of human capital. Hence, the left-hand side of (40) is the after-tax-and-subsidy marginal product of human capital in goods production, measured as physical goods. On the right-hand side, the second term is the subsidy per human capital devoted to human capital production, measured as physical goods. The part denoted by (a) is the marginal product of human capital in human capital production, measured as units of human capital. Hence, the term $A\mu/\lambda$ is interpreted as the unit value of human capital, so that (40) equates the *values* of marginal products of human capital in two production sectors after taxes, subsidies and depreciations.

One can denote the unit value of human capital at period t by $V(t) \equiv A(t)\mu(t)/\lambda(t)$. Since μ/λ is constant on the balanced growth path, V grows at g_A. Then, I can rearrange (40) as

$$\left[1 - \alpha\tau_K - (1-\alpha)\tau_L - \frac{\alpha s u}{1-u} - s\right](1-\alpha)(K/Y)^{\alpha/(1-\alpha)} = V_0 B_0 \phi h_0^{\phi+\theta-1} u^{\phi-1},$$

where I introduce a new constant, $V_0 \equiv V(0)$, so that $V(t) = e^{g_A t} V_0$. Using this, I can rewrite (12) as

$$\rho + \sigma\left(g_C - g_L\right) = \left[1 - \alpha\tau_K - (1-\alpha)\tau_L - \frac{\alpha s u}{1-u}\right] \frac{(1-\alpha)(K/Y)^{\alpha/(1-\alpha)}}{V_0} + g_A - \delta_h.$$

A.2. Foregone Income due to Human Capital Production

First, consider the foregone income due to schooling. Using the census data, McMahon (1991) computes it by average earnings of a worker with the next lower educational level. His Appendix B provides year-1987 estimates. For elementary students, the foregone income is negligible. For secondary students, it is about the same as their total expenditure on education. For post-secondary students, it is about 70 percent as high as their total expenditure on education. Also, Table 25 of Digest of Education Statistics, published by the U.S. Department of Education, suggests that the total expenditure for education in elementary and secondary schools is about 4.0 percent of GDP in 1987. For colleges and universities, it is 2.4 percent of GDP.[26] Using these numbers, I conclude that the foregone income of all students becomes roughly 3.7 percent of GDP. A similar level of foregone income is also reported in Clotfelter, Ehrenberg, Getz and Siegfried (1991).

[26] Since 1987 is in the middle of years 1960-2005, this is a fair estimate for the average over 1960-2005.

Second, consider the foregone income due to on-the-job training. From the wage profiles that Rosen (1982) estimates, Mincer (1994) computes a fraction of earnings capacity devoted to on-the-job training to be 8.5 percent.[27] Recall that non-educational human capital income is 63.6 percent of GDP. Hence, if all on-the-job training hours were to be used for goods production, it would generate additional human capital income worth 5.9 percent ($= 8.5 \times 63.6/91.5$) of GDP.

[27] See Table 7 of Mincer (1994). Also, Mulligan (1998) discusses the estimates of the fraction of time spent in on-the-job training, according to experience levels, based on the panel-data analysis of Heckman, Lochner and Taber (1998). Mulligan (1998) also provides the survey result conducted by Survey Research Center in 1976. These are roughly consistent with the number provided by Mincer (1994).

Table 1. Calibration Result

(A) Values Used to Solve the Equation System, (15) through (21)

Notation	Description	Value
α	K share in Y production	0.34
δ_h	Depreciation rate of H	3.5%
$\delta_K(K/Y)$	Consumption of fixed capital over Y	0.12
g_A	Productivity growth in Y production	1.5%
g_h	Growth of H per worker	0.7%
g_L	Population growth	1.1%
g_Y	Growth of Y	3.3%
i	Investment rate	0.21
s	Subsidy rate to H production	0.15
τ_K	Tax rate on K income in Y production	0.43
τ_L	Tax rate on H income in Y production	0.25
u	Fraction of H devoted to H production	0.28

(B) Values Calibrated from the Solution to Equation System, (15) through (21)

Notation	Description	Value
$Bh_0^{\phi+\theta-1}$	Productivity in H production	0.08
δ_K	K depreciation rate	4.4%
g_B	Productivity growth in H production	0.2%
ϕ	Power coefficient for direct input	0.28
θ	Power coefficient for externalities	0.44
K/Y	K/Y ratio	2.79
r	Before-tax-and-subsidy interest rate	12.3%
r^*	After-tax-and-subsidy interest rate (See Note.)	9.0%
$r^* - \delta_K$	Net after-tax-and-subsidy interest rate (See Note.)	4.6%
V_0	Unit value of H when $A = 1$	11.36

Note: ρ and σ are not calibrated. From (10), define an "after-tax-and-subsidy interest rate" to be $r^* \equiv \left[1 - \alpha\tau_K - (1-\alpha)\tau_L + \frac{(1-\alpha)su}{1-u}\right] r$. The "net after-tax-and-subsidy interest rate," $r^* - \delta_K$, is equated to $\rho + \sigma\left(g_C - g_L\right)$ in the Euler equation.

////////// TABLE 1 CONTINUED //////////

(C) Sensitivity Check on θ

(Original Calibrated Value: θ =0.44)

Alternative value	Remark	Calibrated value
$g_A = 0$	Purely endogenous model	θ =0.73
$g_B = 0$	Minimum g_B	θ =0.72
$g_B = 0.4\%$	Maximum g_B	θ =0.16
$\delta_h = 2.2\%$	Retirement only	θ =0.41
$\delta_h = 1.3\%$	Individual depreciation only	θ =0.36

Table 2. Social Rate of Return on Human Capital

Concept	Notation	Estimation
(A) Private rate of return on H after δ_h and g_A	$r^* - \delta_K$	4.6%
(B) Private rate of return on H before δ_h and g_A (= Private marginal product of H)	(A)+$\delta_h - g_A$	6.6%
(C) Social rate of return on H (= Social marginal product of H)	(B)+$\theta B_0 h_0^{\phi+\theta-1} u^{\phi}$	9.0%

Table 3. National Accounts with Human Capital Production
(GDP in the Current National Account = 100.)

Account #1: Currently adopted.

Account #2: Includes foregone human capital income in human capital production. Includes the gains from learning externalities. Uses the private value of human capital, $V_0 = 11.36$.

Account #3: Uses the social value of human capital, $V_0 = 17.57$.

(A) Product Account

Concept	#1	#2	#3
Total production (=(Aa)+(Ab))	100	139	164
(Aa) Y production, Y (=(Ba)+(Bb1))	93	93	93
(Ab) H production, $VB[Auh]^{\phi}\overline{h}^{\theta}L$ (=(Bb2)+(Bc))	7	46	71

(B) Income Account

Concept	#1	#2	#3
Total income (=(Ba)+(Bb)+(Bc))	100	139	164
(Ba) Physical capital income, $\alpha Y = rK$	32	32	32
(Bb) Human capital income (=(Bb1)+(Bb2))	68	87	101
(Bb1) in Y production, $(1-\alpha)Y$	61	61	61
(Bb2) in H production, $(1-\theta)VB[Auh]^{\phi}\overline{h}^{\theta}L$	7	26	40
(Bc) Gain from externalities, $\theta VB[Auh]^{\phi}\overline{h}^{\theta}L$	0	20	31

(C) Capital Stock Account

Concept	#1	#2	#3
Total stock (=(Ca)+(Cb))		1,125	1,598
(Ca) Physical capital stock, K	261	261	261
(Cb) Human capital stock, VhL		864	1,337

Note: (Aa) is GDP (normalized to 100) minus human capital income in educational services and scientific R&D services. (Ab) in Account #1 is human capital income in educational services and scientific R&D services. (Ab) in Accounts #2 and #3 adds the foregone income due to schooling and on-the-job training. (Ba) is GDP minus human capital income. (Bb1) is a fraction $1-\alpha$ of (Aa). In Account #1, (Bb2) is the same as (Ab). In Accounts #2 and #3, (Bb2) is a fraction $1 - \theta$ of (Ab), and (Bc) is a fraction θ. See the text for (Cb).

Table 4. Calibration Results based on Alternative Assumptions

(A) $g_A = \frac{2}{3}(g_Y - g_L)$

Return on H in H production multiplied by:	1 (benchmark)	$\frac{2}{3}$	$\frac{1}{2}$
θ	0.44	0.25	0.05
Difference between social and private returns	2.3%	1.3%	0.3%

(B) $g_A = 0$

Return on H in H production multiplied by:	1	$\frac{2}{3}$	$\frac{1}{2}$
θ	0.73	0.66	0.58
Difference between social and private returns	5.0%	4.4%	3.9%

Table 5. Average Earnings of High School Graduates and College Graduates

(National Average Earnings=100)

Age	30	40	50	60
High School Graduates	77	90	95	93
College Graduates	109	150	167	170

Source: Census of population, Earnings by Occupation and Education. Reported numbers are on average of the observations in 1980, 1990 and 2000.

Table 6. Calibration Results, with a Common Pool of Teachers

	$g_A = 0$	$g_A = \frac{2}{3}(g_Y - g_L)$
θ (benchmark estimate multiplied by 2/3)	0.49	0.29
Difference between social and private returns	3.3%	1.6%

Table 7. A Teacher's Wage Relative to Per-Capita GDP

Years	1960s	1980s
Data	2.33	1.76
Data (1960s= 100)	100	76
Model, reflecting changes in taxes and subsidies (1960s= 100)	100	96
Model, reflecting changes in taxes and subsidies and constant human capital of a teacher (1960s= 100)	100	83

Figure 1. Output Convergence, 1963-2005

(A) U.S. States

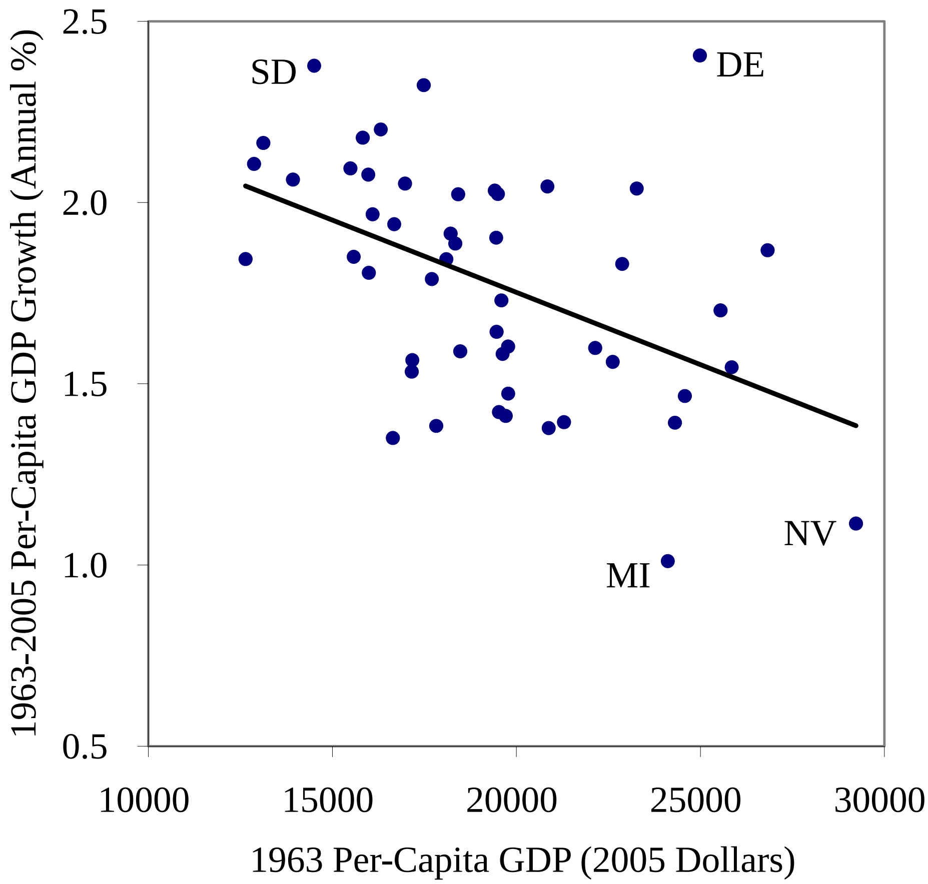
INSERT FIGURE 1 PART (A) HERE.

(B) West European Countries

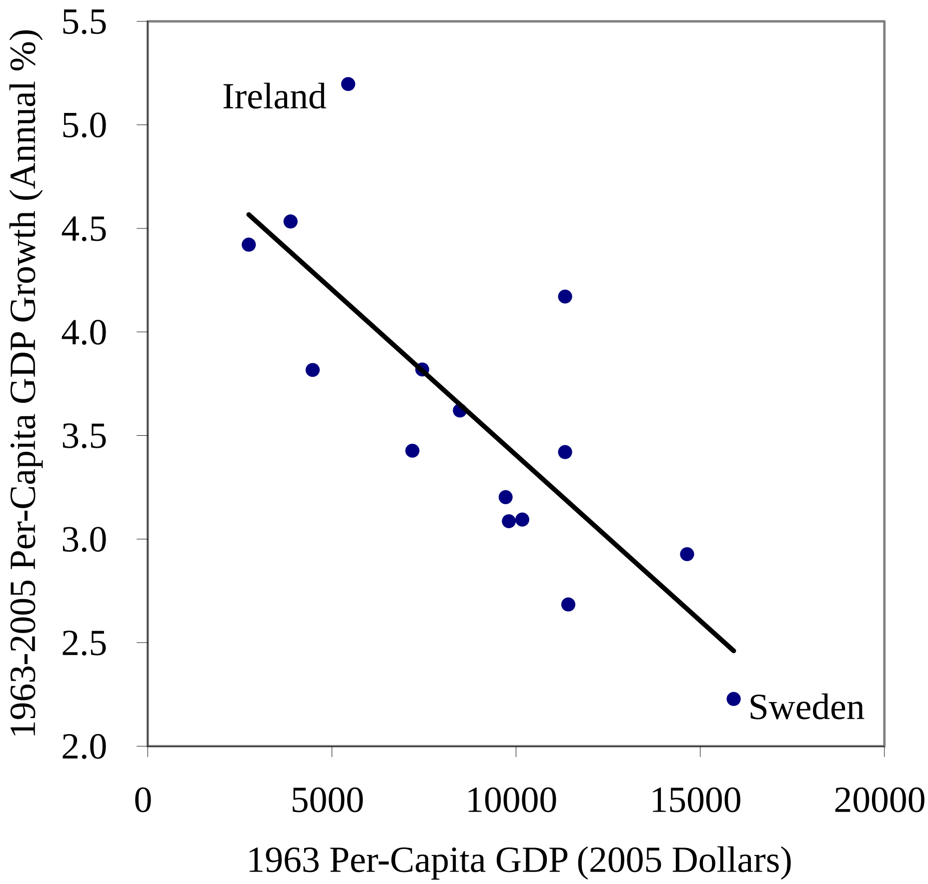
INSERT FIGURE 1 PART (B) HERE.

Note: The West European Countries are Austria, Belgium, Denmark, Finland, France, Italy, Netherlands, Norway, Sweden, Switzerland, United Kingdom, Greece, Portugal, Spain, and Ireland.

PLEASE ADD THIS AS FIGURE 1 PART (A)



PLEASE ADD THIS AS FIGURE 1 PART (B)



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