Online Appendix

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In this Online Appendix, we provide robustness checks on the quantitative results of the model. We discuss the issues related to the calibrations reported in Section III of the paper as well.

A. Robustness Checks

1. Risk Aversion (γ): According to the theory, risk aversion plays an important role in generating the momentum effect. We assumed $\gamma = 3.5$ in our baseline calibration with dividend data. To understand the quantitative effect of γ, Part (A) of Table O1 reports the calibration results with $\gamma = 10$, while other parameter values remain the same as in Table 1. According to Part (A) of Table O1, as the risk aversion increases from 3 to 10, the risk-free asset is in greater demand, and hence, the risk-free rate ($r^f(s_t)$) decreases. On the other hand, the equity is in less demand, and hence, the equity return ($E[r^A_{t+1}|s_t]$ and $E[r^B_{t+1}|s_t]$) increases.

Importantly, as γ increases, the equity return react more sensitively to the changes in s_t. The gap between the two expected returns on A and B, $E[r^B_{t+1}|s_t] - E[r^A_{t+1}|s_t]$, is more than 7% when $\gamma = 10$, while it was smaller when $\gamma = 3.5$. This is because when $s_t = 0.1$, the the correlation between asset B's dividend growth and consumption growth is high, and hence, a more risk-averse investor needs a greater compensation (i.e., higher expected return) to hold asset B. Recall from the discussion on the sources of the momentum effect, it is expected that the effect becomes stronger as the risk aversion increases. Part (A) of Table O1 states that it indeed strengthens the momentum effect.

2. Parameters α and ε: We assumed $\alpha = 0$ and $\varepsilon = 0$ in the baseline case. To better understand the roles of α and ε on the momentum effect, Part (B) of Table O1 reports the calibration results with alternative parameter values for α and ε, while other parameter values remain the same as in Table 1.[1] To provide the result first, as the values of α and ε change, the expected normalized momentum profit remains comparable to the observed profit. This is because the sources of the momentum effect, theoretically discussed in Subsection II.C, are not directly affected by α or ε.

We consider two values for ε, that is, $\varepsilon = 0$ (zero impact of aggregate shock on dividend shock) and 0.45 (positive impact of aggregate shock on dividend shock). As discussed in Subsection III.A, the value of α does not matter when $\varepsilon = 0$. For $\varepsilon = 0.45$, we consider two values of α, that is, $\alpha = -0.45$ (negative autocorrelation of aggregate shock) and 0.45 (positive autocorrelation). A combination of $\varepsilon = 0.45$ and $\alpha = -0.45$ provides an autocorrelation of consumption growth of -0.76. On the other hand, a combination of $\varepsilon = 0.45$ and $\alpha = 0.45$ provides an autocorrelation of 0.76.[2] One can view that our sensitivity check considers very strong autocorrelations, both positive and negative to reflect that GDP and aggregate dividend growth rates are positively and negatively autocorrelated, respectively.

In Part (B) of Table O1, the expected normalized momentum profits change as the values assumed for ε and α change. However, it still remains substantial, at 1.7 cents (when $\varepsilon = 0.45$ and $\alpha = -0.45$) or 1.2 cents (when $\varepsilon = 0.45$ and $\alpha = 0.45$). This is within the observed levels of the normalized momentum profit, between 0.6 and 2.6 cents, previously reported in Part (B) of Table 1.

B. Using the GDP Data

The calibrations so far have used the dividend data with relative risk aversion $\gamma = 3.5$. Here, we use the GDP data to calibrate a model with two identical assets as in Subsection II.B. Since the GDP growth is less volatile than aggregate dividend growth, using the same level of γ is still subject to the equity premium puzzle. (See, for example, Mehra and Prescott (2003).) Since this paper does not focus on resolving these puzzles, we assume a relatively higher value for γ to make reasonable quantitative predictions on the

[1]We performed a similar exercise in Section III.C using the Jegadeesh-Titman strategy. However, due to the different rules used to form portfolios, it is hard to isolate the effect of α and ε in this case.

[2]In our model, α and ε jointly determine the autocorrelation of consumption growth.

equity premium and risk-free rate. We assume $\gamma = 57.5$ following the estimation of Chen, Favilukis and Ludvigson (2011).

The data for GDPs are obtained from the National Income and Product Account Table 1.1.5. As we did for dividend growth, we subtract population growth and inflation to obtain real per-capita GDP growth. The real per-capita GDP growth constructed in this way has an average of 2.1% and a volatility of 5.4% in 1953-2008. To match these, we use $\mu = 1.021$ and $\sigma = 0.054$.

Real per-capita GDP growth and real per-capita dividend growth have a correlation coefficient of 0.43. The hypothesis that the correlation is zero is rejected with a p-value of 0.001.[3] However, these two growth rates have autocorrelations of opposite signs. Real per-capita GDP growth has an autocorrelation of 0.55. However, real per-capita dividend growth has an autocorrelation of -0.38. This implies that while these two growth rates co-move, they may be exposed to additional noise components.

Part (A) of Table O2 summarizes the calibration result with $\alpha = 0$ and $\varepsilon = 0$. Hence, this table serves as a counterpart to the baseline calibration reported in Part (B) of Table 1. The asset returns are 7.0% (when $s_t = 0.5$) or up to 7.5% (when $s_t = 0.1$), which is close to the data. The expected normalized momentum profit can be as high as 1.2 cents (when $s_t = 0.1$), which is comparable to observed levels of $0.6 \sim 2.6$ cents. The Sharpe ratio is 0.19 (when $s_t = 0.1$) which is also comparable to observed levels. This implies that an asset-pricing model calibrated with GDP data, which can generate reasonable equity returns and risk-free rates, can also generate reasonable momentum profits.

Part (B) of Table O2 reflects alternative values assumed for α and ε (as we considered in Part (B) of Table O1). When $\varepsilon = 0.45$ and $\alpha = -0.45$, the autocorrelation of consumption growth becomes -0.76 while the expected normalized momentum profit becomes 2.3

[3] The real per-capita dividend growth is also positively correlated with real per-capita consumption growth from Sydney Ludvigson's website. The correlation coefficient is 0.30 with a p-value of 0.027. In addition, it is positively correlated with real per-capita non-durable consumption growth (constracted based on the same dataset as GDPs). The correlation coefficeint is 0.46. The p-value is 0.0003. Robert Shiller's website provides aggregate dividends data for the stocks listed in S&P 500. The correlation is 0.36, while the p-value is 0.007.

cents. When $\varepsilon = 0.45$ and $\alpha = 0.45$, the autocorrelation of consumption growth becomes 0.76 while the expected normalized momentum profit becomes 0.3 cents. These profits are comparable to the observed levels of $0.6 \sim 2.6$ cents.

It should be noted that in both Part (B) of Table O1 (using aggregate dividend data) and Part (B) of Table O2 (using GDP data), the autocorrelation of consumption growth changes from -0.76 to 0.76 depending on the assumed values for ε and α, but the expected normalized momentum profits remain sizable. Hence, the assumption about the autocorrelation of aggregate shock does not substantially affect the mechanism of the momentum effect.

C. Size Effect, Value Effect, and Long-Term Reversal

In Figure 2, as s_t increases, the expected return on asset A mostly increases. This implies that an asset's size is positively associated with the expected return. Is the model consistent with the size effect? The model implication seems inconsistent with the empirical observation that small-cap stocks tend to provide higher average returns. However, the cross-sectional differences in expected returns can be generated not only by the differences in consumption shares but also by the differences in the dividend processes. In Subsection III.B, we allowed the dividend processes to differ across the assets. Indeed, for Case 1, when $s_t = 0.5$, we had $E[r^A_{t+1}|s_t] = 13.2\%$ and $E[r^B_{t+1}|s_t] = 7.4\%$ (not reported in Table 3), where asset A is the "small" portfolio and asset B is the "big" portfolio. Hence, The model calibrated with the data on dividends provides the results consistent with the size effect. In addition, the value effect, i.e., a stylized fact that the stocks with higher book-to-market ratio tend to provide higher average returns, can be generated in our model as well. For Case 2, when $s_t = 0.5$, we had $E[r^A_{t+1}|s_t] = 6.9\%$ and $E[r^B_{t+1}|s_t] = 9.7\%$ (not reported in Table 3), where asset A is the "low-book-to-market" portfolio and asset B is the "high-book-to-market" portfolio.[4]

[4] We do not claim that our model can explain the size and value effects. However, the quantitative results suggest that cross-sectional differences in dividend processes and shares matter to account for these anomalies. This is in line with the recent literature mentioned earlier.

Finally, we discuss if the model is able to generate the long-term reversal, which is also an important stylized fact in the momentum literature. Figure O1 illustrates $E[s_{t+1}|s_t] - s_t$, according to s_t. The value of $E[s_{t+1}|s_t] - s_t$ is positive when $0 < s_t < 0.5$ and negative when $0.5 < s_t < 1$, implying a mean reversion to 0.5. This is a possible source of the long-term reversal. If asset A has a positive dividend shock and asset B has a negative dividend shock when $s_t = 0.5$, then s_{t+1} rises from s_t, and the momentum effect arises. However, $\{s_t\}$ tends to revert back to 0.5, and hence (as Figure 2 suggests) the expected return on asset A will decrease. This implies that a positive dividend shock at t is related to a higher expected return at $t+1$, which is a source of momentum, but at the same time, it is related to a lower expected return in the long run, which can be viewed as a source of the long-term reversal.

References

Chen, X., J. Favilukis, and S. C. Ludvigson. "An Estimation of Economic Models with Recursive Preferences," Yale University Working Paper (2011).

Mehra, R., and E. C. Prescott, "The Equity Premium in Retrospect," In *Handbook of the Economics of Finance*, G. Constantinides, M. Harris, and R. Stulz, eds. Cambridge, MA: Elsevier (2003), 887–936.

Table O1: Robustness Checks ($s_t = 0.1$)

Part (A) of this table presents an update of Part (B) of Table 1 with $\gamma = 10$. Part (B) of this table represents an update of Part (B) of Table 1 with alternative values for $\alpha$and ε.All other parameters are the same as Part (A) of Table 1.

(A) Risk Aversion (γ)

γ	3.5	10
Expected momentum profit (%)	0.1	0.2
Sharpe ratio	0.09	0.18
Expected normalized momentum profit (¢)	2.6	4.0
Sharpe ratio	0.15	0.22
$E[r^A_{t+1}\|s_t]$ (asset A's return) (%)	4.7	6.1
$E[r^B_{t+1}\|s_t]$ (asset B's return) (%)	9.4	13.2
$r^f(s_t)$ (risk-free rate) (%)	-1.6	-7.3

(B) α and ε

ε	0	0.45	0.45
α	0	-0.45	0.45
Expected momentum profit (%)	0.1	0.0	0.0
Sharpe ratio	0.09	0.03	0.01
Expected normalized momentum profit (¢)	2.6	1.7	1.2
Sharpe ratio	0.15	0.16	0.09
Autocorrelation of consumption growth	0	-0.76	0.76

Table O2: Baseline Calibration with GDP Data

Part (A) of this table is an update of Part (B) of Table 1, with $g_H^A = g_H^B = 1.075$and $g_L^A = g_L^B = 0.967$ (which are parametrization based on GDP data), as well as $\gamma = 57.5$. All other parameters are the same as Part (A) of Table 1. Part (B) considers alternative values for $\alpha$and ε.

(A) Baseline: $\alpha = 0$ and $\varepsilon = 0$

	Data	$s_t = 0.1$	$s_t = 0.5$
Expected momentum profit (%)	(0.0, 0.2)	0.0	0.0
Sharpe ratio	(0.08, 0.21)	0.15	0.01
Expected normalized momentum profit (¢)	(0.6, 2.6)	1.2	0.1
Sharpe ratio	(0.07, 0.23)	0.19	0.01
$E[r_{t+1}^A \vert s_t]$ (asset A's return) (%)	7.9	5.2	7.0
$E[r_{t+1}^B \vert s_t]$ (asset B's return) (%)		7.5	7.0
$r^f(s_t)$ (risk-free rate) (%)	0.6	1.9	1.8
Average of consumption growth (%)	2.1	2.1	2.1
Volatility of consumption growth (%)	5.4	4.9	3.8

(B) Changes in α and ε to Part (A)

ε	0	0.45	0.45
α	0	-0.45	0.45
Expected momentum profit (%)	0.0	0.0	0.0
Sharpe ratio	0.15	0.61	0.01
Expected normalized momentum profit (¢)	1.2	2.3	0.3
Sharpe ratio	0.19	0.86	0.08
Autocorrelation of consumption growth	0	-0.76	0.76

Figure O1: Mean Reversion of $\{s_t\}$

This figure illustrates $E\left[s_{t+1}|s_t\right] - s_t$, according to s_t, implied by the model described in Subsection II.A.